

693.7

C123b

nr. 2

cop. 2

FOUAD I UNIVERSITY - GIZA

FACULTY OF ENGINEERING.

REINFORCED CONCRETE SECTION

Bulletin 2.

**DIMENSIONING AND CHECKING OF STRESSES
OF R. C. RECTANGULAR SECTIONS
UNDER NORMAL STRESSES**

By

MICHEL BAKHOUM, B. Sc. (Hons)

Demonstrator, Faculty of Engineering.

CAIRO

COSTA TSOUMAS & Co. PRESS

FOUAD I UNIVERSITY - GIZA

FACULTY OF ENGINEERING.

REINFORCED CONCRETE SECTION

Bulletin 2.

**DIMENSIONING AND CHECKING OF STRESSES
OF R. C. RECTANGULAR SECTIONS
UNDER NORMAL STRESSES**

By

MICHEL BAKHOUM, B. Sc. (Hons)

Demonstrator, Faculty of Engineering.

THE LIBRARY OF THE

FEB 19 1948

UNIVERSITY OF ILLINOIS

CAIRO

COSTA TSOUMAS & Co. PRESS.

693.7

C 1232

no. 2

cop. 2

CONTENTS

	PAGE
Introduction.	1
Notations	3

PART 1. Theoretical Review

Introduction	4
(A) Determination of the Reinforcement for a section of given Depth	6
Condition for Minimum Reinforcement	8
(B) Design of Sections with given Ratio of $A'_s : A_s$	11
Special case : $A'_s = 0$	15
(C) Check of Stresses	17

PART 2. Summary and Applications

(A) Procedure and Examples on Determination of the Reinforcement for a Section of given Depth	18
(B) Procedure and Examples on Design of Sections with given Ratio of $A'_s : A_s$	21
(C) Procedure and Examples on the Check of Stresses.	26

1948.9. Engineering, no 2-3, 1940 - 44 cont. - Catechism 5 Apr 48

INTRODUCTION

This investigation is a study of some of the problems with which designers have to deal in the dimensioning or checking of stresses of R.C rectangular sections under normal stresses. It is restricted to the case in which the external force or couple lies on an axis of symmetry of the section. The case of double bending or eccentric normal forces acting outside the axes of symmetry has been treated in another paper by the author and submitted to the Faculty in November 1940. The point of application of the normal force is also considered to lie outside the ideal core of the section. The other case where the resultant falls within that core represents no difficulty and can be treated as a homogeneous section.

The following points are treated for the cases of simple bending, eccentric compression and eccentric tension.

(A) Determination of the required areas of tensile and compressive reinforcements for a section of given depth and determination of the stresses which make the area of steel in the section to be minimum.

(B) Determination of the required areas of tensile and compressive reinforcements if the ratio of the compressive to tensile reinforcement is given.

(C) Check of stresses.

The formulae which give the required unknowns in the three cases are complicated. The solution is simplified by providing direct curves and tables, all of which are unitless and attempted to be put in the most possible general form. For any special conditions, special curves or tables can be obtained by substitution in the general formulae or in the given tables and curves.

The first part of the paper deals with the theoretical review and deduction of curves and the second part with applications and examples on the use of the curves and tables. The second part is so arranged that it can be read separately.

The writer wishes to acknowledge his great indebtedness to Prof. Dr. W. S. Hanna Head of the R. C. Dept. and to Prof. Dr. Schwyzer Head of the Theory of Structures and Metallic Construction Dept. at the Faculty of Engineering, Fouad 1 University for their most valuable advices.

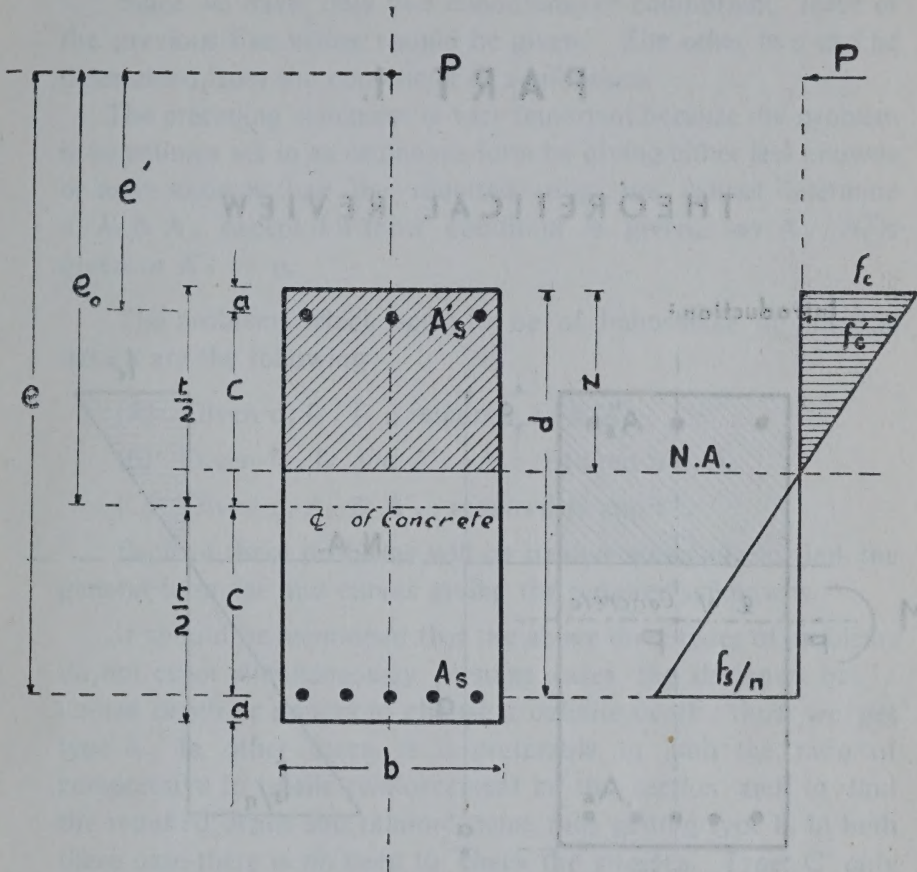
December 1940

M. B.

LIST OF TABLES & CURVES

CURVE	TABLE	PURPOSE
1	1	Determination of the Reinforcement for a Section of given Depth under Bending, Eccentric Comp. or Tension.
2	2	Condition for Minimum Reinforcement
3	3	Design of Sections under Simple Bending for given Ratio $A'_s : A_s$
4 to 9	4 to 9	Design of Sections under Eccentric Comp. or Tension for $A'_s : A_s = 0; 0.2; 0.4; 0.6; 0.8; 1$
10	10	Design of Sections with no Comp. Reinforcement under Bending; Eccentric Comp. or Tension
11 to 16	—	Check of Stresses under Bending Eccentric Compression or Tension.

NOTATIONS



- P = External normal force acting on axis of symmetry of the section.
 $M = P e_o$ = Moment of P about the horizontal axis at middle of the section.
 $M_s = P e$ = Moment about tensile reinforcement.
 $M'_s = P e'$ = Moment about compressive reinforcement.
 M_1 = Moment about centre of action of compression.
 n = Ratio of the modulus of elasticity of steel to that of concrete.
 $\mu = A_s / b d$ = Ratio of area of tensile reinforcement to effective area of concrete.
 $\mu' = A'_s / b d$ = Ratio of area of compressive reinforcement to effective area of concrete.
 $\alpha = A'_s / A_s$ = Ratio of compressive to tensile reinforcement.
 $i = a / d$
 $s = z / d$
 $k = c / d$
 f_c = Maximum compressive stress in concrete.
 f'_c = Unit stress in concrete at compressive reinforcement.
 f_s = Maximum tensile stress in tensile reinforcement.
 $R = f_s : f_c$

PART I.

THEORETICAL REVIEW

Introduction :

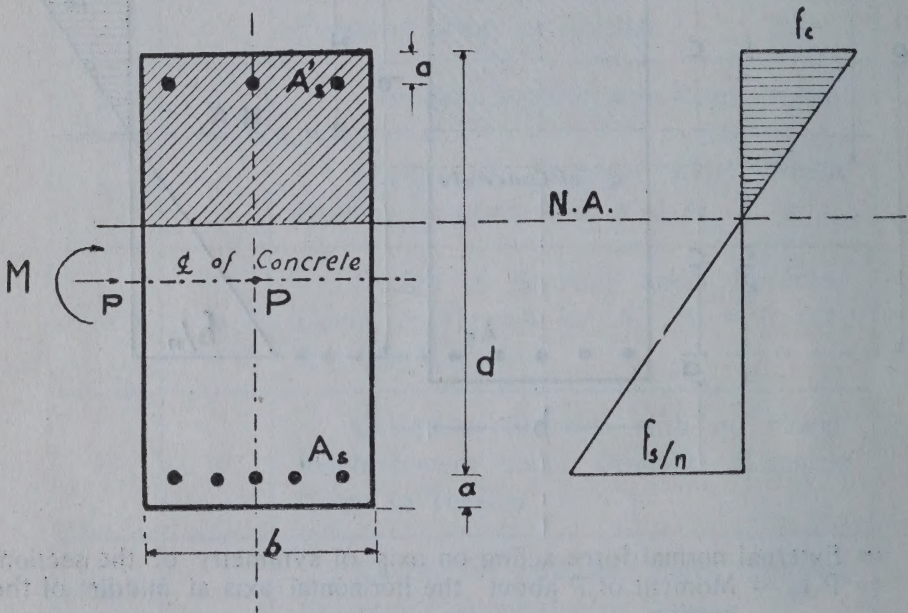


Fig. (1)

The items which enter in the design or the check of stresses of a R.C. rectangular section under a normal force P and a couple M are the following (fig. 1) :

M, P — f_c, f_s, n — b, d, A_s, A'_s, a

From these , M & P are given values,

b & a are chosen values,

and n can be considered as a constant value.

Thus we are left with the following arbitrary values :

$f_c, f_s, d, A_s, \text{ \& } A'_s.$

Since we have only two conditions of equilibrium, three of the previous five values should be given. The other two can be determined from the conditions of equilibrium.

The preceding statement is very important because the problem is sometimes set in an erroneous form by giving either less knowns or more knowns than the required. e. g. we cannot determine d , A_s & A'_s except if a third condition is given, say A'_s / A_s is given or $A'_s = 0$.

The problems which seem to be of importance in practical design are the following:

- (A) Given d , f_c , f_s ; required A_s & A'_s .
- (B) Given f_c , f_s and A'_s / A_s ; required d & A_s .
- (C) Given d , A_s & A'_s ; required f_c and f_s .

Each of these problems will be treated separately to find the general formulae and curves giving the required unknowns.

It should be mentioned that the above three types of problems do not occur simultaneously. In some cases the designer has a limited depth or prefers to choose a definite depth ; thus we get type A. In other cases, it is preferable to limit the ratio of compressive to tensile reinforcement in the section and to find the required depth and reinforcement, thus getting type B. In both these case there is no need to check the stresses. Type C only occurs when both the depth and the reinforcement have been preliminarily chosen without design according to (A) or (B) and it is required to compute the maximum resulting stresses.

(A) Given : The depth of a R. C. rectangular section, subjected to a couple or to an eccentric compressive or tensile force and the values of maximum stresses.

Required : determination of tensile and compressive reinforcement.

Referring to fig. (2), (3), (4) the following equations apply to each of the three cases :

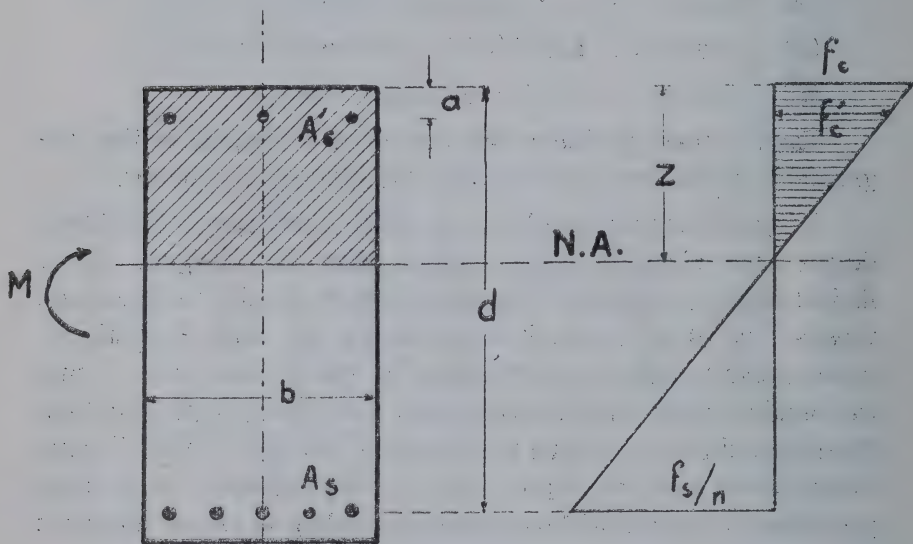


Fig. (2)

$$z = n f_c d / (n f_c + f_s) = n d / (n + R) \quad \text{where } R = f_s / f_c$$

$$= s d \quad \text{where } s = n / (n + R)$$

$$f'_c = f_c (z - a) / z \quad \text{and putting } a = i d$$

$$f'_c = f_c (1 - i / s)$$

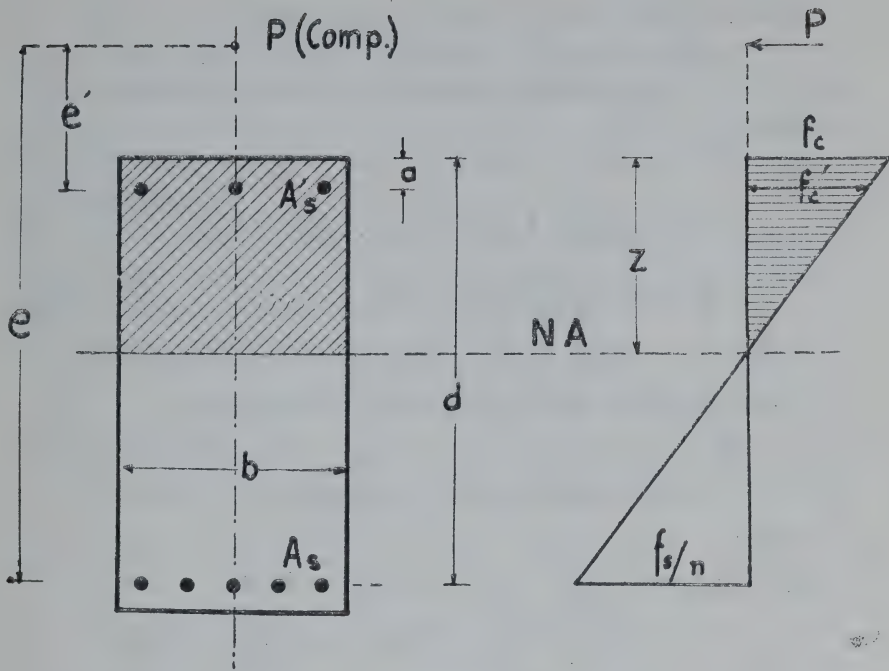


Fig. (3)

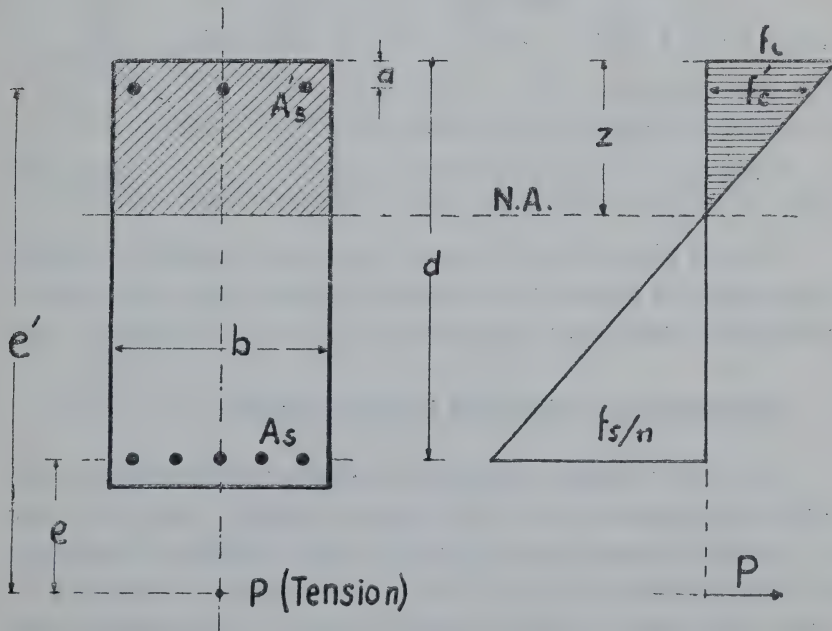


Fig. (4)

Taking moments about tensile reinforcement:

$$\begin{aligned} M_s &= b f_c z (d - z / 3) / 2 + n A'_s f'_c (d - a) \\ &= b f_c s d (d - s d / 3) / 2 + n \mu' d f_c (1 - i / s) (d - i d) \end{aligned}$$

Thus:

$$\begin{aligned} M_s / b d^2 f_c &= s (1 - s / 3) / 2 + n (1 - i / s) (1 - i) \mu' \\ \mu' &= \frac{M_s}{b d^2 f_c} \left(\frac{s}{n (s - i) (1 - i)} \right) - \left(\frac{s^2 (3 - s)}{6 n (s - i) (1 - i)} \right) \\ \mu' &= C_1 \left(\frac{M_s}{b d^2 f_c} \right) - C_2 \quad \dots \dots \dots (1) \end{aligned}$$

where $C_1 = s / n (s - i) (1 - i)$ & $C_2 = s^2 (3 - s) / 6 n (s - i) (1 - i)$

Taking moments about compressive reinforcement:

$$\begin{aligned} M'_s &= A_s f_s (d - a) - b f_c z (z / 3 - a) / 2 \\ &= \mu b d f_s (d - i d) - b f_c s d (s d / 3 - i d) / 2 \end{aligned}$$

Thus:

$$\begin{aligned} M'_s / b d^2 f_c &= \mu R (1 - i) - s (s / 3 - i) / 2 \\ \mu &= \frac{M'_s}{b d^2 f_c} \left(\frac{1}{R (1 - i)} \right) + \left(\frac{s (s - 3i)}{6 R (1 - i)} \right) \\ \mu &= C_3 \left(\frac{M'_s}{b d^2 f_c} \right) + C_4 \quad \dots \dots \dots (2) \end{aligned}$$

where $C_3 = 1 / R (1 - i)$ & $C_4 = s (s - 3i) / 6 R (1 - i)$

For simple bending $M'_s = M_s = M$

For eccentric compression or tension $M_s = P e$ & $M'_s = P e'$

Equations (1) and (2) give the values of μ' & μ . The values of C_1, C_2, C_3 and C_4 are given in table (1).

Curve 1 gives directly μ and μ' for cases of bending, eccentric compression or eccentric tension and for any values of f_s & f_c . Both table 1 and curve 1 are constructed for $n = 15$ and $i = 0.08$

Condition for Minimum Reinforcement:

It is not necessary that the minimum reinforcement in the section corresponds to the maximum permissible values of f_c and f_s . It will be seen that in some cases of eccentric compression the minimum value of $(A'_s + A_s)$ corresponds to values of f_s which are lower than the permissible values. The problem thus becomes as follows:—

Given d, f_c

Required f_s, A_s, A'_s such that $A'_s + A_s = \text{minimum}$.

It should be noted that the reduction of one of the knowns is accompanied by the addition of the extra condition of minimum reinforcement.

Solution :

For minimum reinforcement, it is clear that the maximum permissible value of f_c should be fully utilised. The value of f_s is to be obtained from the condition :

$$\frac{d}{df_s} (A_s + A'_s) = 0$$

From equation (1):

$$\begin{aligned} \mu' &= \frac{M_s}{bd^2 f_c} \left(\frac{s}{n(s-i)(1-i)} \right) - \left(\frac{s^2(3-s)}{6n(s-i)(1-i)} \right) \\ n(1-i)\mu' &= \frac{M_s}{bd^2 f_c} \left(\frac{s}{s-i} \right) - \left(\frac{s^2(3-s)}{6(s-i)} \right) \quad \dots (a) \end{aligned}$$

$$\text{where } s = n / (n + R) \text{ \& } R = \frac{f_s}{f_c}$$

From equation (2):

$$\mu = \frac{M'_s}{bd^2 f_c} \left(\frac{1}{R(1-i)} \right) + \left(\frac{s(s-3i)}{6R(1-i)} \right)$$

Putting $R = n(1-s)/s$

$$n(1-i)\mu = \frac{M'_s}{bd^2 f_c} \left(\frac{s}{1-s} \right) + \left(\frac{s^2(s-3i)}{6(1-s)} \right) \quad \dots (b)$$

Differentiating equations (a) & (b) with respect to f_s and adding :

$$\begin{aligned} n(1-i) \frac{d}{df_s} (\mu' + \mu) &= \\ \frac{M_s}{bd^2 f_c} \left(\frac{-i}{(s-i)^2} \right) \frac{ds}{df_s} &- \left(\frac{3s^2 - 2s^3 - 6is + 3is^2}{6(s-i)^2} \right) \frac{ds}{df_s} \\ + \left(\frac{3s^2 - 2s^3 - 6is + 3is^2}{6(1-s)^2} \right) \frac{ds}{df_s} &= 0 \end{aligned}$$

$$\text{Since } \frac{ds}{df_s} = \frac{-n}{f_c(n+R)^2} \neq 0$$

Thus:

$$\begin{aligned} & \frac{M_s}{bd^2 f_c} \left(\frac{-i}{(s-i)^2} \right) + \frac{M'_s}{bd^2 f_c} \left(\frac{1}{(1-s)^2} \right) \\ & + \left(\frac{(3s^2 - 2s^3 - 6is + 3is^2)(i-1)(1+i-2s)}{6(1-s)^2(s-i)^2} \right) = 0 \\ & \left(\frac{M_s}{bd^2 f_c} \right) - \left(\frac{M'_s}{bd^2 f_c} \right) \left(\frac{(s-i)^2}{i(1-s)^2} \right) \\ & + \left(\frac{(3s^2 - 2s^3 - 6is + 3is^2)(1-i)(1+i-2s)}{6i(1-s)^2} \right) = 0 \quad \dots (3) \end{aligned}$$

Equation (3) gives the value of R which fulfils the condition that $(A_s + A'_s) = \text{minimum}$. Values of R can be obtained from table (2) or curve (2) which is constructed for $n = 15$ and $i = 0.08$

(B) Given: The ratio of compressive to tensile reinforcement for a R. C. rectangular section subjected to a couple or to an eccentric compressive or tensile force.

Required: determination of the depth and reinforcement.

(a) Simple Bending: (Fig. 5)

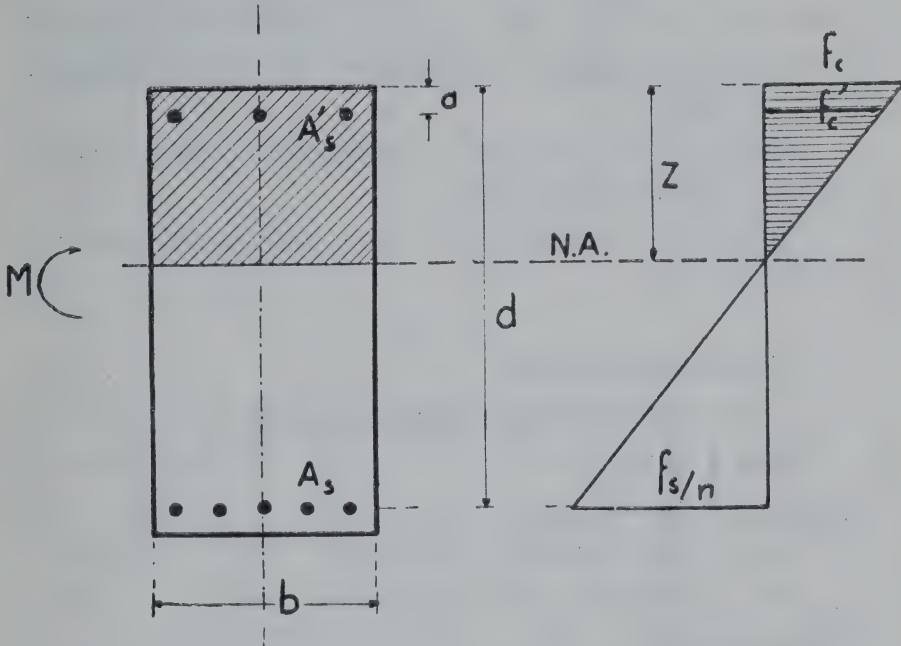


Fig. (5)

Given $A'_s / A_s = \alpha$

Equations of equilibrium are:

$$A_s f_s = b f_c z / 2 + n f_c A'_s \quad (i)$$

$$M = b f_c z (d - z / 3) / 2 + n f_c A'_s (d - a) \quad (ii)$$

we have $z = sd$ where $s = n / (n + R)$ & $R = f_s / f_c$

$$\& f'_c = f_c (1 - a / z) = f_c (1 - i / s)$$

Thus from equation (i):

$$A_s f_s = b f_c sd / 2 + n f_c (1 - i / s) \alpha A_s$$

$$A_s = bdsf_c / 2 (f_s - \alpha n f_c (1 - i / s))$$

$$\mu = \frac{s}{2(R - \alpha n(1 - i/s))} \quad \& \quad \mu' = \alpha \mu \quad (4)$$

and from equation (ii):

$$\begin{aligned} M &= (b d s f_c / 2) (d - s d / 3) + n f_c (1 - i / s) \alpha \mu (d - i d) b d \\ M / b d^2 f_c &= s (1 - s / 3) / 2 + \alpha n \mu (1 - i / s) (1 - i) \\ d^2 &= M / f_c b (s (1 - s / 3) / 2 + \alpha n \mu (1 - i / s) (1 - i)) \\ d &= K_1 \sqrt{\frac{M}{b}} \dots \dots \dots (5) \end{aligned}$$

where $K_1 = c / \sqrt{f_c}$ & $c = 1 / \sqrt{s(1-s/3)/2 + \alpha n \mu (1-i/s)(1-i)}$

Equation (4) determines the reinforcement and equation (5) determines the depth.

Values of μ & c are given for different ratios of R in tables 3 (a) and 3 (b) and in curves (3).

Note: The same tables and curves can also be used for check of stresses.

(b) Eccentric Forces.

(i) Eccentric Compression: (fig. 6)

Given $A'_s / A_s = \alpha$

Equations of Equilibrium:

$$P = b f_c z / 2 + n f'_c A'_s - A_s f_s \quad (i)$$

$$M'_s = A_s f_s (d - a) - b f_c z (z / 3 - a) / 2 \quad (ii)$$

From equation (i):

$$P = b f_c s d / 2 + n f_c (1 - i / s) \alpha \mu b d - \mu b d f_s$$

$$P / b d f_c = s / 2 + \alpha \mu n (1 - i / s) - \mu R$$

Thus:

$$d = \left(\frac{P}{b f_c} \right) \frac{1}{\gamma} \dots \dots \dots (6)_a$$

Where $\gamma = s / 2 + \alpha \mu n (1 - i / s) - \mu R$

From equation (ii):

$$P e' / b d^2 f_c = \mu R (1 - i) - (s / 2) (s / 3 - i)$$

but

$$\begin{aligned} P e' / b d^2 f_c &= P (e_o - k d) / b d^2 f_c \\ &= P b^2 f_c \gamma^2 (e_o - k p / b f_c \gamma) / b f_c P^2 \\ &= b f_c e_o \gamma^2 / P - k \gamma \end{aligned}$$

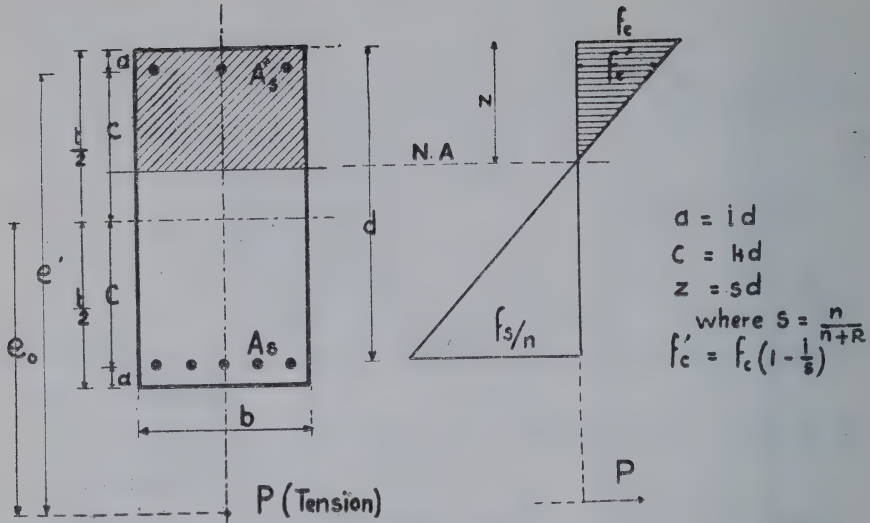


Fig. (7)

Thus :

$$d = \left(\frac{P}{b f_c} \right) \frac{1}{\gamma} \dots \dots \dots (6)_b$$

where $\gamma = \mu R - s/2 - \alpha \mu n (1 - i/s)$

From equation (ii):

$$P e' / b d^2 f_c = \mu R (1 - i) - (s/2) (s/3 - i)$$

$$\begin{aligned}
 P e' / b d^2 f_c &= P (e_o + k d) / b d^2 f_c \\
 &= b f_c e_o \gamma^2 / P + k \gamma
 \end{aligned}$$

Thus:

$$b f_c e_o \gamma^2 / P + k \gamma = \mu R (1 - i) - (s/2) (s/3 - i)$$

Substituting for $\gamma = \mu R - s/2 - \alpha \mu n (1 - i/s)$,

$$\begin{aligned}
 \frac{b f_c e_o}{P} \left(\mu R - \frac{s}{2} - \alpha \mu n \left(1 - \frac{i}{s} \right)^2 - k \left(\mu R - \frac{s}{2} - \alpha \mu n \left(1 - \frac{i}{s} \right) \right) \right) \\
 = \mu R (1 - i) - \frac{s}{2} \left(\frac{s}{3} - i \right) \dots \dots \dots (7)_b
 \end{aligned}$$

It can be noticed that equations (6)_b & (7)_b for eccentric tension are identical with equations (6)_a & (7)_a for eccentric compression.

Curves (4) to (9) and tables (4) to (9) give the depth and reinforcement of rectangular sections subjected to eccentric compressive or tensile forces for ratios $\alpha = 0, .2, .4, .6, .8, 1$ and for $a = .08d$ ($k = .46, i = .08$) and $n = 15$.

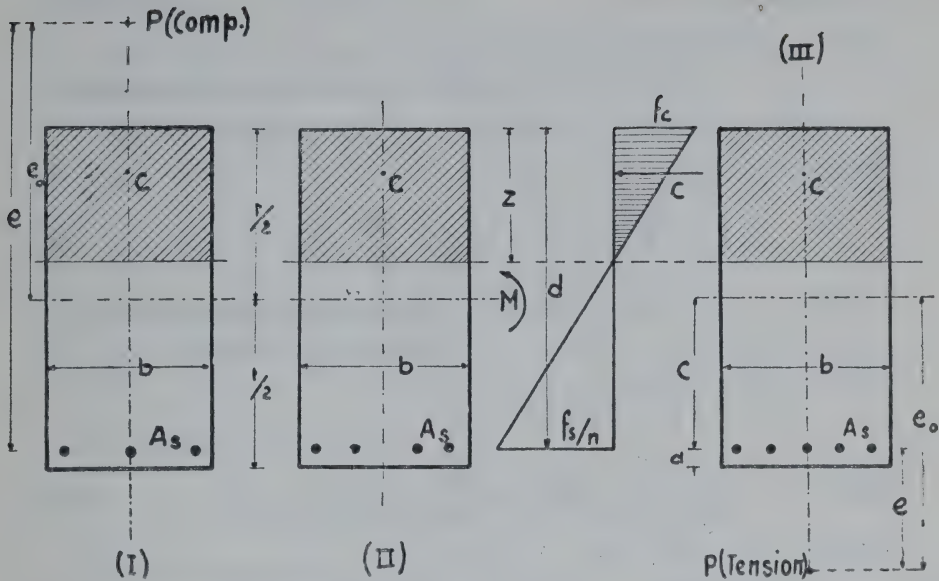


Fig. (8)

Special Case: Sections with no compression reinforcement under bending ; eccentric compression and eccentric tension. (Fig. 8)

For each of the three cases shown in (fig. 8) the following equations apply :

$$bf_c z (d - z/3) / 2 = M_s \quad (i)$$

$$\text{and } A_s f_s (d - z/3) = M_1 \quad (ii)$$

where M_1 = Moment about the centre of compression.

From equation (i) :

$$bf_c s d (d - s d / 3) / 2 = M_s \quad \text{where } s = n / (n + R)$$

$$\text{then } d = \sqrt{6 / s (3 - s) f_c} \times \sqrt{M_s / b}$$

$$d = K_1 \sqrt{\frac{M_s}{b}} \quad \dots \dots \dots (8)$$

$$\text{where } K_1 = c / \sqrt{f_c} \quad \text{and } c = \sqrt{6 / s (3 - s)}$$

When applying equation (8) in cases of eccentric compression or eccentric tension, the depth should be preliminarily assumed, because M_s is a function of d . An alternative to this is to refer

to the centre of the section instead of referring to the tensile reinforcement. Thus:

$$d = K_1 \sqrt{M_s / b}$$

$$= K_1 \sqrt{P (e_o \pm kd) / b} \quad + \text{ for eccentric compression}$$

$$d^2 / K_1^2 = M / b \pm P kd / b \quad - \text{ for eccentric tension.}$$

Thus :

$$d = K_1 \left(\pm r + \sqrt{r^2 + \frac{M}{b}} \right) \dots \dots \dots (9)$$

where $r = P k K_1 / 2 b$ and $+$ for eccentric compression
 $-$ for eccentric tension.

From equation (ii):

$$A_s f_s d (1 - s / 3) = M_1$$

Thus

$$A_s = \frac{M_1}{K_2 d} \dots \dots \dots (10)$$

where $K_2 = c' f_s$ and $c' = 1 - s / 3$

$$\text{or } A_s f_s d (1 - s / 3) = M_s \pm P (1 - s / 3)$$

Thus :

$$A_s = \frac{M_s}{K_2 d} \pm \frac{P}{f_s} \dots \dots \dots (11)$$

$-$ for eccentric compression

$+$ for eccentric tension.

For determining the depth either equation (8) or (9) can be used and for determining the reinforcement either equation (10) or (11). Values of c and c' required for determining K_1 and K_2 are given in table (10) and curves (10) for $n = 15$.

(C) Check of stresses under simple bending, eccentric compression or tension.

Referring to figures (2), (3), (4), the following equations apply for each of the three cases :

$$M_s = Pe = bf_c z (d - z / 3) / 2 + nf'_c A'_s (d - a) \quad (i)$$

$$M'_s = Pe' = A_s f_s (d - a) - bf_c z (z / 3 - a) / 2 \quad (ii)$$

and we have : $z = sd$ where $s = n / (n + R)$

$$f'_c = f_c (1 - i / s)$$

Substituting in equations (i) and (ii) we get :

$$Pe / bd^2 f_c = s (1 - s / 3) / 2 + \alpha \mu n (1 - i / s) (1 - i) \quad (iii)$$

$$Pe' / bd^2 f_c = \mu R (1 - i) - s (s / 3 - i) / 2 \quad (iv)$$

From equations (iii) and (iv) :

$$\frac{e'}{e} = \frac{\mu R (1 - i) - s (s - 3i) / 6}{s (3 - s) / 6 + \alpha \mu n (1 - i) (1 - i / s)} \quad (12)$$

and from equation (iii) :

$$f_c = \frac{Pe}{bd^2} \left(\frac{1}{s (1 - s / 3) / 2 + \alpha \mu n (1 - i) (1 - i / s)} \right)$$

or

$$f_c = \left(\frac{Pe}{bd^2} \right) \frac{1}{K} \quad (13)$$

Equation (12) contains R only as unknown. Curves (11) to (16) are computed from this equation for $n = 15$, $a = .08d$ and for values of $\alpha = 0, .2, .4, .6, .8, 1.0$.

Equations (13) gives f_c . Values of K can be taken from curve (1).

For minimum reinforcement from curve (2) $R > 24$

We take $R = 24$ i.e. $f_s = 1200 \text{ kg/cm}^2$ and $f_c = 50 \text{ kg/cm}^2$

From curve (1) or table (1) :

$$\mu = .95 \% , \quad \mu' = .95 \% , \quad \mu' + \mu = 1.90 \%$$

Note : The compression reinforcement can be decreased by increasing the tensile reinforcement and in this case f_s will not be fully utilised and $(\mu' + \mu)$ will be bigger, e. g. :

(1) If μ is taken 1.2% , from curve (1) $R = 19.5$
i.e. $f_s = 975 \text{ kg/cm}^2$ $\mu' = .76 \%$, $\mu' + \mu = 1.96 \%$

(2) If f_s is taken 800 kg/cm^2 i.e. $R = 16$,
 $\mu = 1.5 \%$, $\mu' = .6 \%$, $\mu' + \mu = 2.1 \%$

(3) If f_s is taken 600 kg/cm^2 i.e. $R = 12$,
 $\mu = 2.08 \%$, $\mu' = .38 \%$, $\mu' + \mu = 2.46 \%$

(4) If no compression reinforcement is used
 $\mu' = 0$ $R = 7$ $f_s = 350 \text{ kg/cm}^2$
 $\mu = 3.6 \%$ $\mu' + \mu = 3.6 \%$

Example (2) : A rectangular section $30 \times 50 \text{ cms}$ is subjected to a bending moment $M = 5.0 \text{ m.t.}$ and an axial compressive force $P = 20 \text{ t.}$ Find μ and μ' if the maximum permissible stresses are $f_c = 50 \text{ kg/cm}^2$ and $f_s = 1200 \text{ kg/cm}^2$

Solution : (fig. 10)

$$e_o = 5/20 = .25 \text{ m.}$$

$$e = .46 \text{ m.} , \quad e' = .04 \text{ m.}$$

$$\frac{M_s}{bd^2 f_c} = \frac{20\,000 \times 46}{30 \times 46^2 \times 50} = .29$$

$$\frac{M'_s}{bd^2 f_c} = .20 \times \frac{4}{46} = .026$$

From curve (2) or table (2), for minimum reinforcement

$$R = 13 \quad \text{i.e.} \quad f_s = 650 \text{ kg/cm}^2$$

From curve (1) or table (1) :

$$\mu = .44 \% , \quad \mu' = .60 \%$$

$$\mu' + \mu = 1.04 \%$$

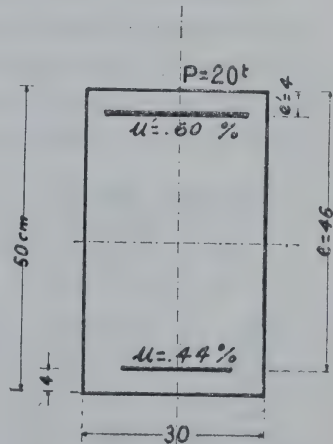


FIG.10

Note : The following table shows that a wide variation of f_s from that giving minimum reinforcement

does not affect the sum of μ and μ' appreciably and hence a wide range of choice can be made.

R	f_c	f_s	μ %	μ' %	$(\mu + \mu')$ %
24	50	1200	.16	1.11	1.27
20	50	1000	.21	.94	1.15
18	50	900	.25	.85	1.10
16	50	800	.30	.75	1.05
14	50	700	.38	.66	1.04
13	50	650	.44	.60	1.04
12	50	600	.49	.55	1.04
10	50	500	.66	.41	1.07
8	50	400	.94	.29	1.23
6	50	300	1.45	.14	1.59
4	50	200	2.54	0	2.54

Example (3) : A rectangular section 40×70 cms is subjected to a moment $M = 20$ m. t. and an axial tensile force $P = 10$ tons. Find μ and μ' if $f_c = 45$ kg/cm² and $f_s = 1300$ kg/cm²

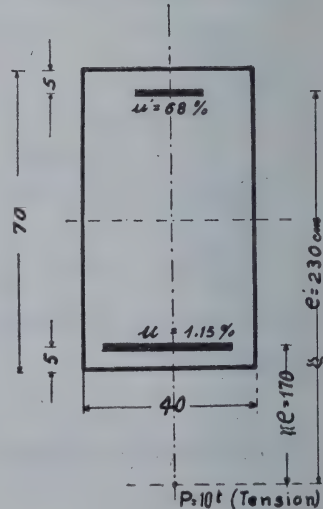
Solution : (fig. 11)

$$e_o = 20/10 = 2.0 \text{ m.}$$

$$e = 1.70 \text{ m.}, \quad e' = 2.30 \text{ m.}$$

$$\frac{M_s}{bd^2 f_c} = \frac{10\,000 \times 170}{40 \times 65^2 \times 45} = .223$$

$$\frac{M'_s}{bd^2 f_c} = .223 \times \frac{230}{170} = .302$$



From curve (1) or table (1) :

$$\mu = 1.15\%, \quad \mu' = .68\%$$

Fig. 11

Example (4): A rectangular section 40×60 cms is subjected to a bending mome n $M = 20$ m.t. Find the required reinforcement if $f_c = 60$ kg/cm² and $f_s = 1200$ kg/cm²

Solution :

$$R = 20$$

$$\frac{M_s}{bd^2 f_c} = \frac{M'_s}{bd^2 f_c} = \frac{2\,000\,000}{40 \times 55^2 \times 60} = .275$$

From table (1) or curve (1) :

$$\mu = 1.56 \% \quad \mu' = .81 \%$$

(B) Given : $\alpha = A'_s / A_s$, f_c and f_s for a section subjected to simple bending, eccentric compression or eccentric tension.

Required : d , A_s & A'_s .

Procedure :

(1) Simple Bending :

From curve (3) or table (3) find μ and then c .

$$\text{then : } K_1 = \frac{c}{\sqrt{f_c}} \quad , \quad d = K_1 \sqrt{\frac{M}{b}}$$

$$A_s = \mu b d \quad , \quad A'_s = \alpha A_s$$

(2) Eccentric compression or tension :

$$(a) \text{ Find the value of } \phi = \frac{bf_c e_o}{P}$$

(b) From curves (4) to (9) or tables (4) to (9), according to the value of α find μ and then γ .

$$\text{then} \quad d = \left(\frac{P}{bf_c} \right) \frac{1}{\gamma}$$

$$A_s = \mu b d \quad , \quad A'_s = \alpha A_s$$

Remark :

In cases of simple bending or eccentric tension the maximum permissible stresses are to be used. For eccentric compression with small eccentricities smaller values of f_s may be tried. Curve (2)

may be used as a guide by assuming a reasonable depth and finding f_s which gives the minimum reinforcement. In any case, however, it is easy to make some alternative designs for the section for different values of R and thus finding the most suitable or the most economical section.

Special Case : Sections with no compression reinforcement.

Instead of using curves (3) to (9) for dimensioning such sections under bending, eccentric compression or tension, the following direct equations can be applied for the three cases :

$$d = K_1 \sqrt{\frac{M_s}{b}}$$

$$\text{or } d = K_1 \left(\pm r + \sqrt{r^2 + \frac{M}{b}} \right)$$

$$\text{where } r = \frac{P}{b} \times \frac{k K_1}{2}$$

+ for eccentric compression

— for eccentric tension.

$$\text{And } A_s = \frac{M_1}{K_2 d}$$

(where M_1 = Moment about centre of compression)

$$\text{or } A_s = \frac{M_s}{K_2 d} \mp \frac{P}{f_s}$$

— for eccentric compression.

+ for eccentric tension.

Values of K_1 and K_2 can be obtained from curve (10) or table (10).

Examples :

Example (1) : It is required to design a rectangular section to carry a bending moment $M = 25.0$ m.t. if $f_c = 60 \text{ kg/cm}^2$, $f_s = 1200 \text{ kg/cm}^2$, $b = 40$ cms and :

$$(i) A'_s : A_s = 0 \quad (ii) A'_s : A_s = 0.5 \quad (iii) A'_s : A_s = 1.$$

Solution : (fig. 12)

$$R = 1200/60 = 20$$

(i) If $A'_s : A_s = 0$, from curve (3) or table (3) we get :

$$c = 2.33 \text{ \& } \mu = 1.07 \%$$

$$\text{then } K_1 = \frac{2.33}{\sqrt{60}} = .30$$

$$d = .30 \sqrt{\frac{25,000}{0.40}} = 75 \text{ cms.}$$

$$A_s = \frac{1.07}{100} \times 40 \times 75 = 32.1 \text{ cm}^2$$

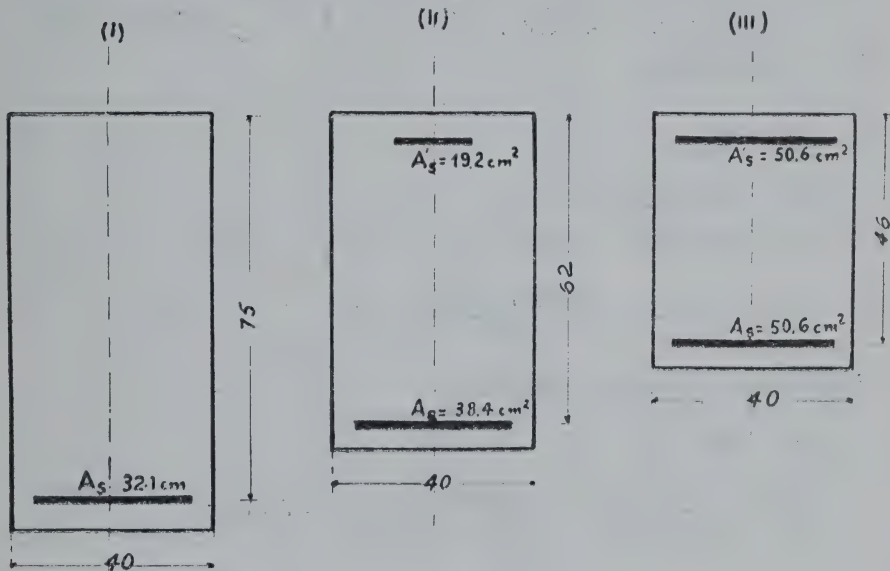


Fig. (12)

(ii) If $A'_s : A_s = .5$

$$c = 1.92 \text{ \& } \mu = 1.55 \%$$

$$K_1 = \frac{1.92}{\sqrt{60}} = .247$$

$$d = .247 \sqrt{\frac{25,000}{0.40}} = 62 \text{ cms}$$

$$A_s = \frac{1.55}{100} \times 62 \times 40 = 38.4 \text{ cm}^2 \quad A'_s = 19.2 \text{ cm}^2$$

(iii) If $A'_s : A_s = 1$

$$c = 1.42 \text{ \& } \mu = 2.75 \%$$

$$K_1 = \frac{1.42}{\sqrt{60}} = .183$$

$$d = .183 \sqrt{\frac{25\,000}{0.40}} = 46 \text{ cms}$$

$$A_s = \frac{2.75}{100} \times 46 \times 40 = 50.6 \text{ cm}^2 \quad A'_s = 50.6 \text{ cm}^2$$

Example (2) : Design a rectangular section to carry a bending moment $M = 20 \text{ m.t.}$ and an axial compressive force $P = 10 \text{ tons}$ if $b = 40 \text{ cms}$, $A'_s : A_s = 1$, $f_c = 50 \text{ kg/cm}^2$ and $f_s = 1200 \text{ kg/cm}^2$.

Solution :

$$R = 1200 / 50 = 24$$

$$e_o = 20 / 10 = 2.0 \text{ m.}$$

$$\phi = \frac{bf_c e_o}{P} = \frac{40 \times 50 \times 200}{10\,000} = 40$$

From curve (9) or table (9) we get :

$$\mu = .94 \% \quad \& \quad \gamma = .078$$

$$d = \left(\frac{P}{bf_c} \right) \frac{1}{\gamma} = \left(\frac{10\,000}{40 \times 50} \right) \frac{1}{.078} = 64 \text{ cms}$$

$$A'_s = A_s = \frac{.94}{100} \times 64 \times 40 = 24.1 \text{ cm}^2$$

Comparisons : (fig. 13)

The following table shows the required depth and reinforcements for some different ratios of $A'_s : A_s$. The table also gives the relative cost and relative weight of the section per unit length considering that for the section without compression reinforcement as unity. In computing the relative cost, the cost of steel, including the manufacture, to that of concrete, including the formwork, per volume is assumed to be 50.

$A'_s : A_s$	μ %	μ' %	$(\mu + \mu')$ %	d	Total A_s	Relative cost	Relative weight
0	.55	0	.55	83	18.3	1	1
0.2	.60	.12	.72	81	23.3	1.04	.97
0.4	.67	.27	.94	79	29.8	1.10	.96
0.6	.74	.44	1.18	74	35.0	1.11	.90
0.8	.82	.65	1.47	69	40.5	1.13	.85
1.0	.94	.94	1.88	64	48.2	1.17	.79

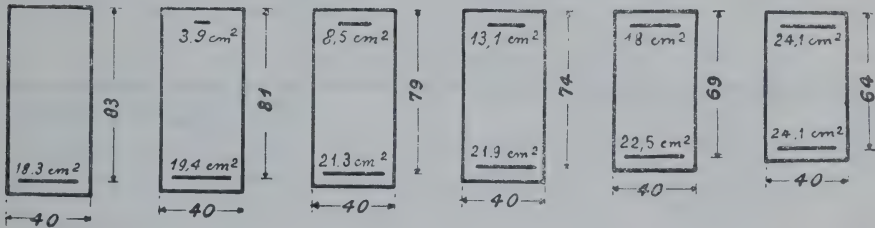


Fig. (13)

Example (3) : It is required to design a rectangular section to carry a tensile centric force $P = 5.0$ tons and a bending moment $M = 12.0$ m.t. if $b = 35$ cms , $A'_s : A_s = 0.5$, $f_c = 45$ kg/cm² and $f_s = 900$ kg/cm²

Solution :

$$R = 900 / 45 = 20 \quad e_o = 12 / 5 = 2.40 \text{ m.}$$

$$\phi = \frac{bf_c e_o}{P} = \frac{35 \times 45 \times 240}{5000} = 75.5$$

From curves (6) & (7) or tables (6) & (7) we get :

$$\mu = 2.07 \% \quad \& \quad \gamma = .072$$

$$d = \left(\frac{P}{bf_c} \right) \frac{1}{\gamma} = \left(\frac{5000}{35 \times 45} \right) \frac{1}{.072} = 44 \text{ cms}$$

$$A_s = \frac{2.07}{100} \times 35 \times 44 = 31.8 \text{ cm}^2$$

$$A'_s = 0.5 \times 31.8 = 15.9 \text{ cm}^2$$

Example (4) : Same as example (2) only P is a tensile force.

The following table shows the required depth and reinforcement and the relative cost and weight of the section for some different ratios of $A'_s : A_s$. The relative volume cost of steel, including manufacture, to that of concrete, including formwork, is assumed to be 50.

$$R = 24 \quad e_o = 2.0 \text{ m.}$$

$$\phi = \frac{b f_c e_o}{P} = \frac{40 \times 50 \times 200}{10,000} = 40$$

$A'_s : A_s$	μ %	μ' %	$(\mu + \mu')$ %	d	Total A_s	Relative cost	Relative weight
0	1.16	0	1.16	58	27	1	1
0.2	1.3	.26	1.56	55.5	34.6	1.07	.97
0.4	1.48	.59	2.07	53.2	44.2	1.18	.93
0.6	1.74	1.04	2.78	49	54.5	1.27	.88
0.8	2.10	1.68	3.78	44.6	67.5	1.41	.82
1.0	2.60	2.60	5.2	40.6	84.5	1.59	.77

(C) Check of stresses under simple bending, eccentric compression or eccentric tension.

Procedure :

Find $\frac{e'}{e}$ or $\frac{e_o}{d}$ and from curves (11) to (16), according to the ratio of $A'_s : A_s$ find the value of R .

From curve (1) find the value of K or K' then :

$$f_c = \frac{M_s}{bd^2 K} \text{ or } f_c = \frac{M'_s}{bd^2 K'}$$

$$f_s = R \cdot f_c$$

Note : For simple bending curve (3) or table (3) may be used instead of using curves (11) to (16).

Examples :

A rectangular section 30×75 cms reinforced with $2 \phi 1''$ on the compression side and $5 \phi 1''$ on the tension side. Concrete cover = 6 cms. (fig. 14). Check the stresses in concrete and steel for the three following cases :

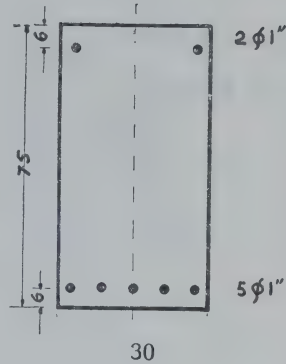


Fig. 14

(i) If it is subjected to a bending moment $M = 16.0$ m.t.

(ii) If it is subjected to a bending moment $M = 16.0$ m.t. and a centric compressive force $P = 20$ tons.

(iii) If it is subjected to a bending moment $M = 16.0$ m.t. and a centric tensile force $P = 20$ tons.

Solution :

$$A_s = 25.0 \text{ cm}^2 \quad A'_s = 10.0 \text{ cm}^2 \quad A'_s : A_s = .4$$

$$\mu = \frac{25}{30 \times 69} = 1.2 \% \quad , \quad \mu' = \frac{10}{30 \times 69} = .48 \%$$

Case (i) : $e' : e = 1$

from curves (13) $f_s : f_c = 21.8$

from curve (1) $K = .23$

$$f_c = \frac{16,000 \times 100}{30 \times 69^2 \times .23} = 48.8 \text{ kg/cm}^2$$

$$f_s = 21.8 \times 48.8 = 1064 \text{ kg/cm}^2$$

Case (ii) : $e_o = 80$ cms, $e = 111$ cms, $e' = 49$ cms

$$e' : e = .44$$

from curve (13) $f_s : f_c = 13.5$

from curve (1) $K' = .125$

$$f_c = \frac{20,000 \times 49}{30 \times 69^2 \times .125} = 55 \text{ kg/cm}^2$$

$$f_s = 13.5 \times 55 = 745 \text{ kg/cm}^2$$

Case (iii) : $e_o = 80 \text{ cms}$, $e = 49 \text{ cms}$, $e' = 111 \text{ cms}$

$$e' : e = 2.27$$

from curve (15) $f_s : f_c = 37$

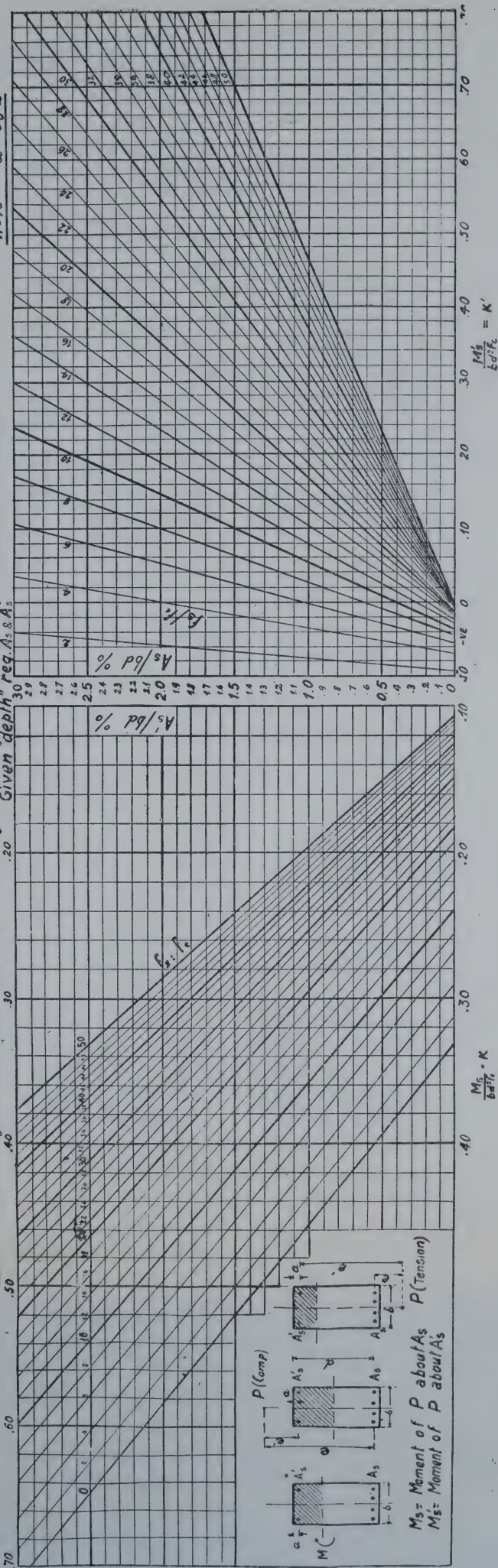
from curve (1) $K' = .42$

$$f_c = \frac{20,000 \times 111}{30 \times 69^2 \times .42} = 37 \text{ kg/cm}^2$$

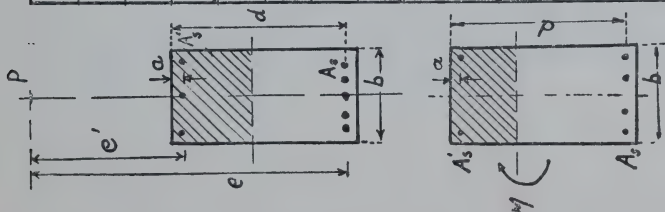
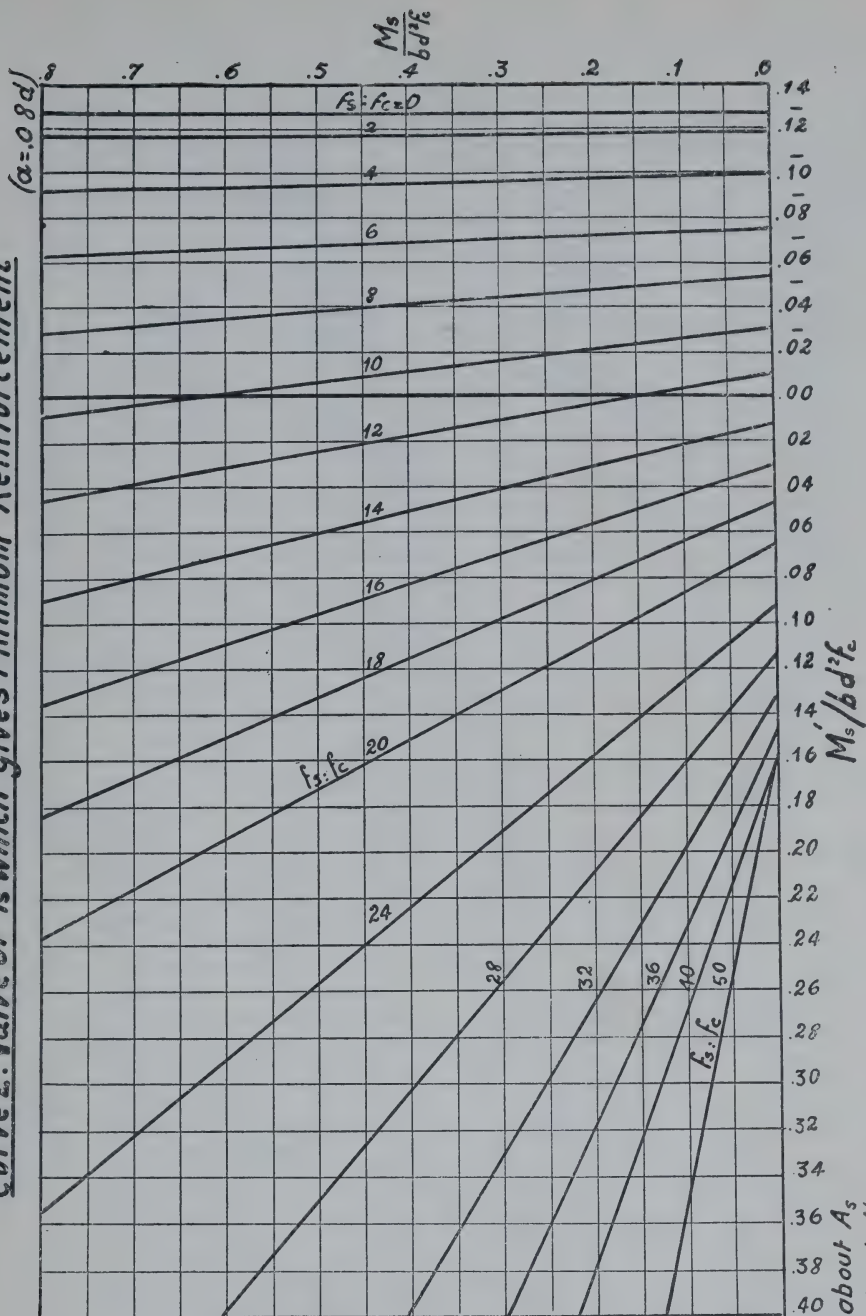
$$f_s = 37 \times 37 = 1370 \text{ kg/cm}^2$$

Curve 1. Rectangular Sections under Bending, Eccentric Compression & Eccentric Tension

$n = 15$ $\alpha = .08d$



Curve 2: Value of f_s which gives Minimum Reinforcement



M_s = Moment about A_s
 M_s' = Moment about A_s'

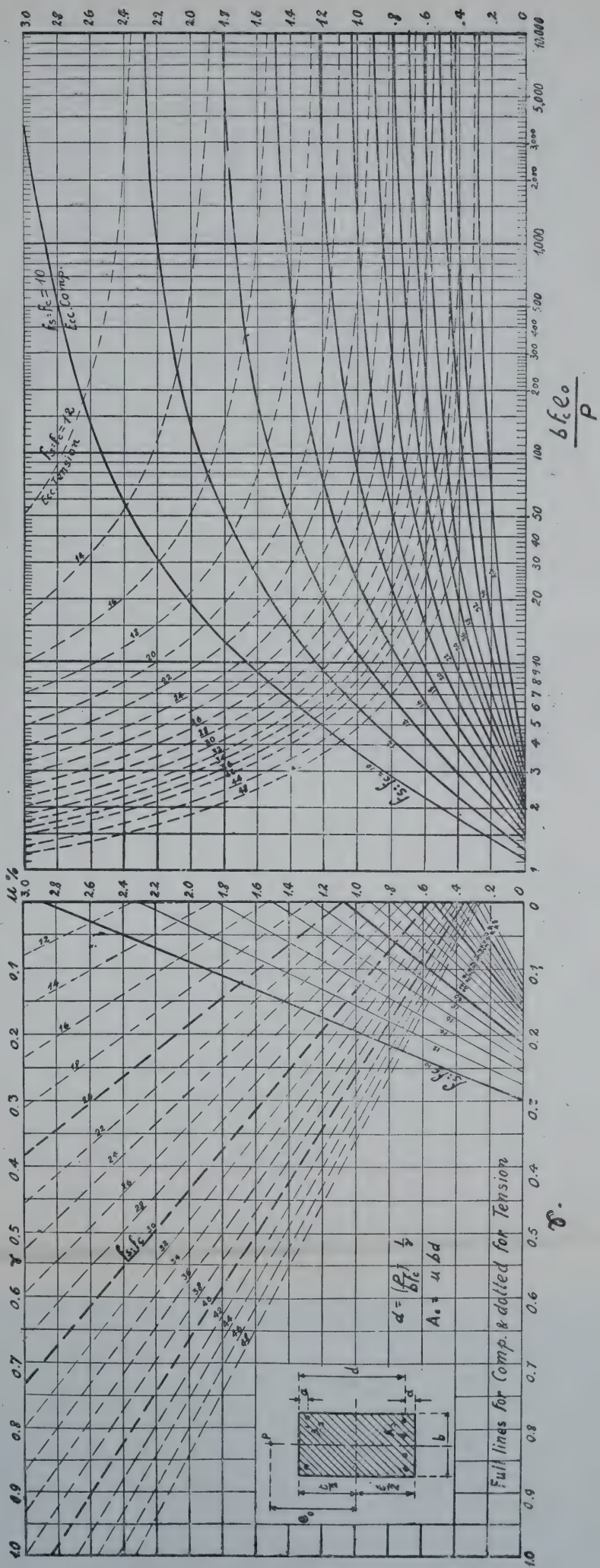
Curve 4

Dimensioning of Rectangular Sections under Eccentric Compression or Tension

$$a = .08d$$

$$n = 15$$

$$A'_s : A_s = 0$$



Curve 5

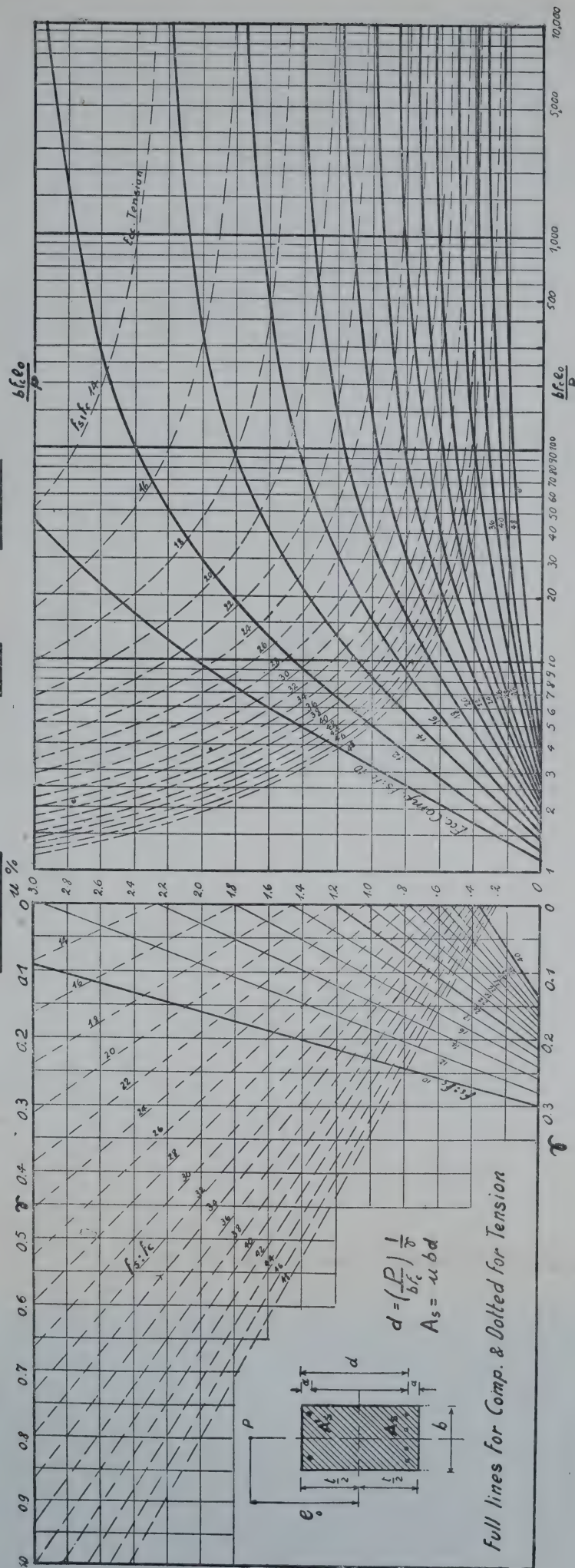
Dimensioning of Rectangular Sections under Eccentric Compression or Tension

$$\underline{A'_5: A_5 = -2}$$

%

$$n = 15$$

$$a = .08d$$

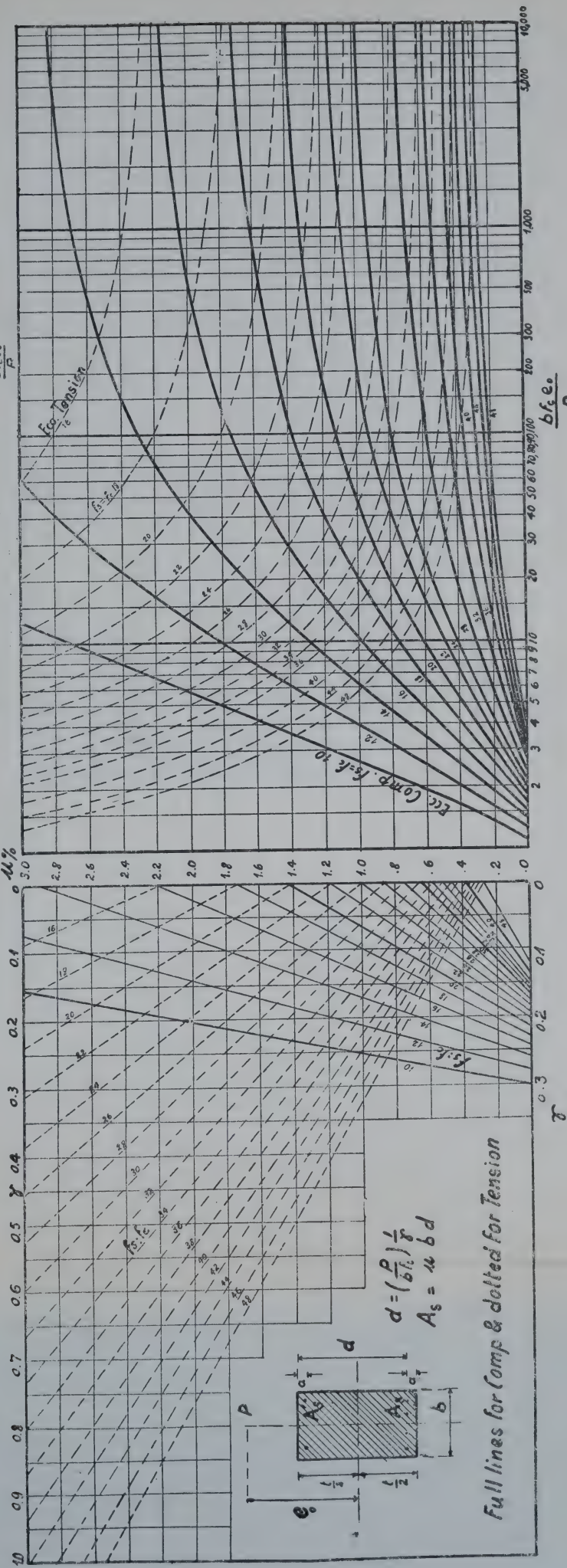


Dimensioning of Rectangular Sections under Eccentric Compression or Tension

$$\frac{n=15}{a=0.8d}$$

$$A'_s = A_s = .4$$

$$\frac{b f_c e}{P}$$



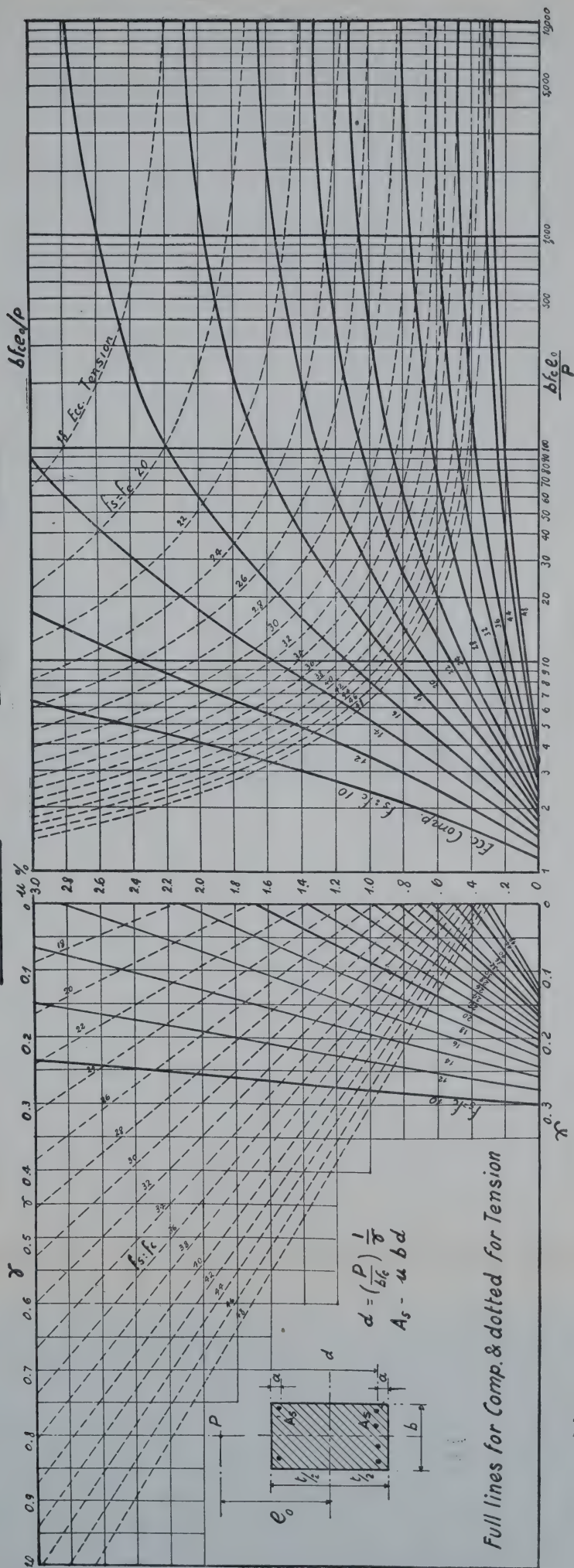
Curve 7

Dimensioning of Rectangular Sections under Eccentric Compression or Tension

$$\underline{A'_3 = A_3 = -6}$$

$$n = 15$$

$$\underline{a = .08d}$$



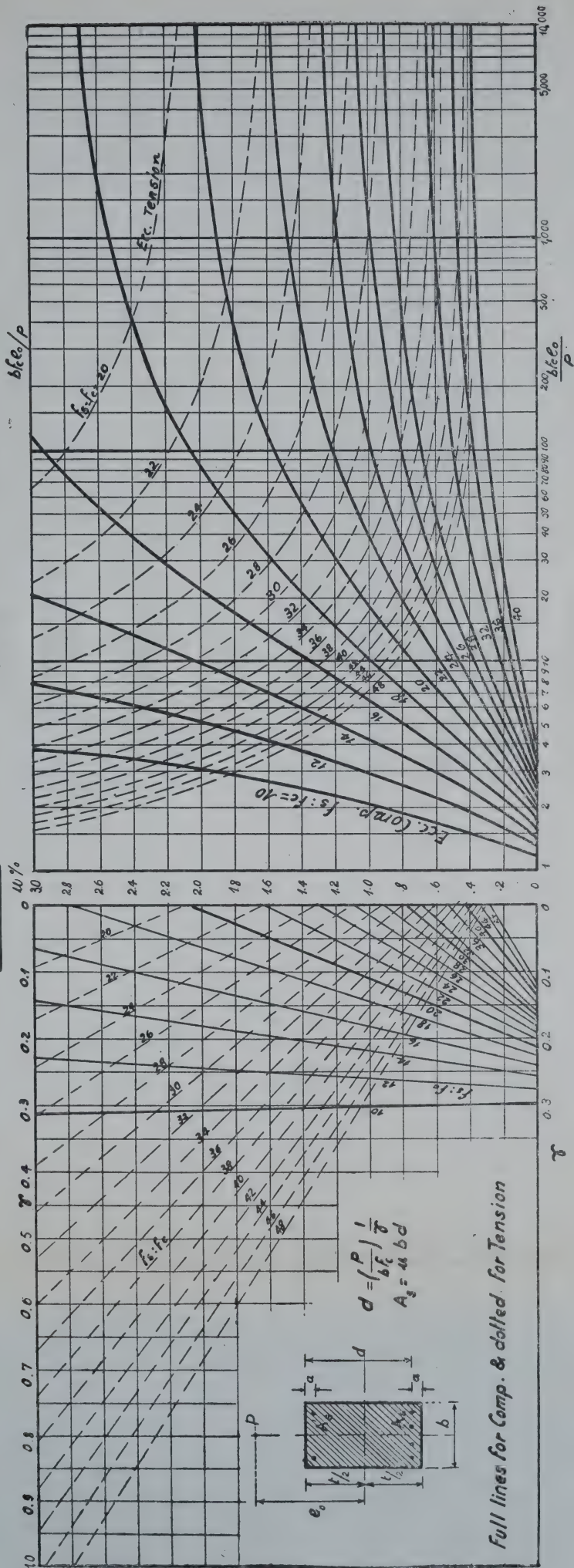
Curve 8

Dimensioning of Rectangular Sections under Eccentric Compression or Tension

$$A'_s : A_s = .8$$

$$n = 15$$

$$a = .08 d$$



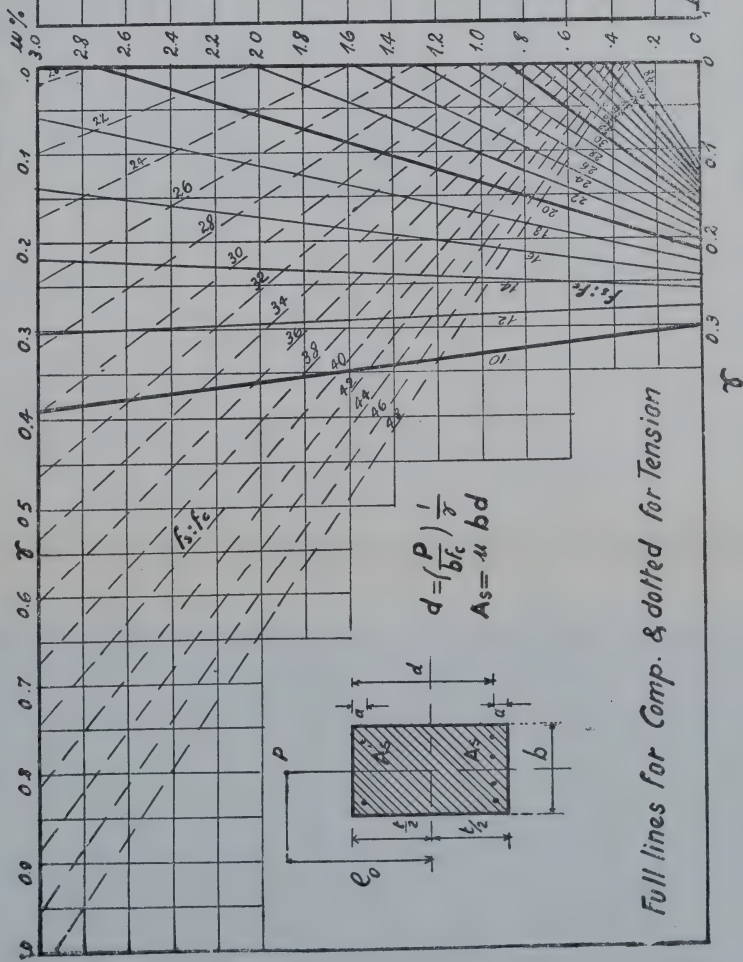
$$A_s' : A_s = 1$$

$$n = 15$$

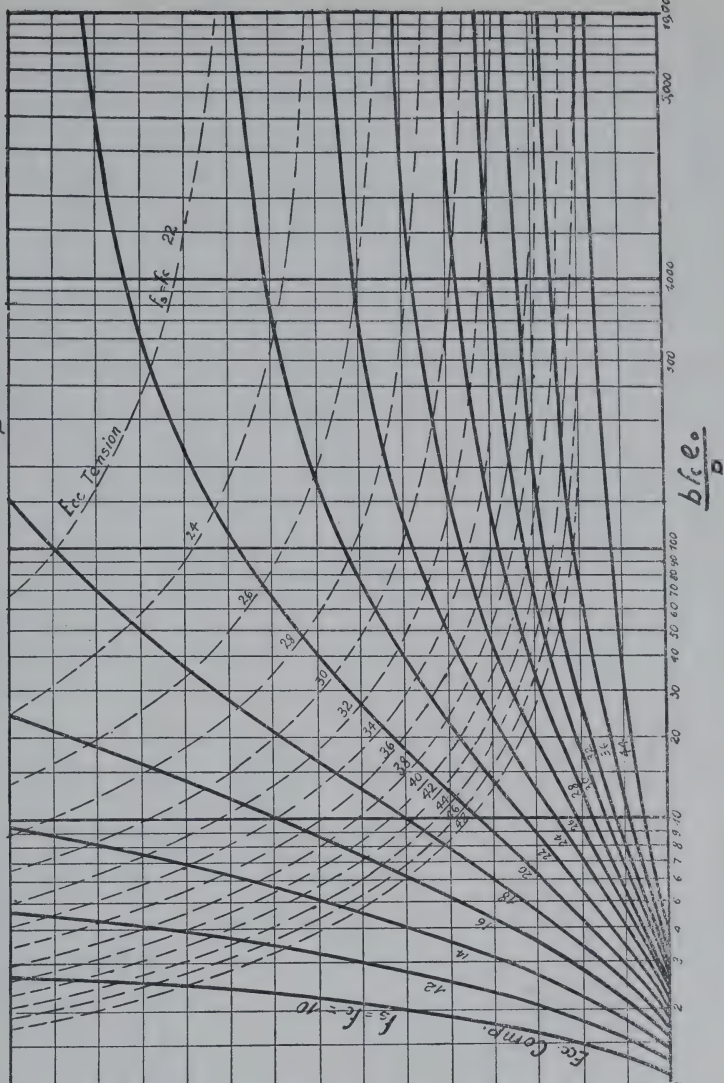
$$a = 0.08d$$

$$\frac{bke_o}{P}$$

$$0.0 W\%$$



Full lines for Comp. & dotted for Tension

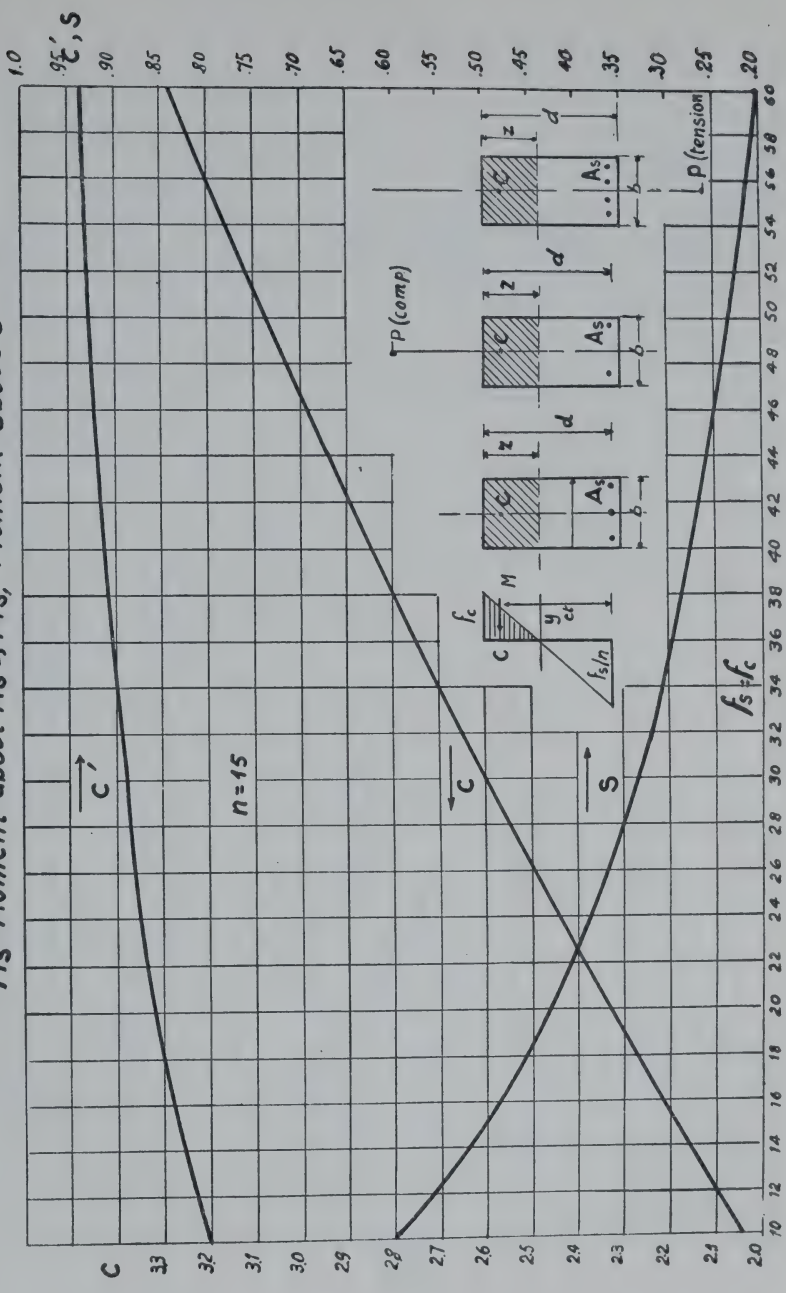


$$\frac{bke_o}{P}$$

Curve: 10-Rectangular Sections with no Compression reinf. under Bending, Eccentric comp. or Tension

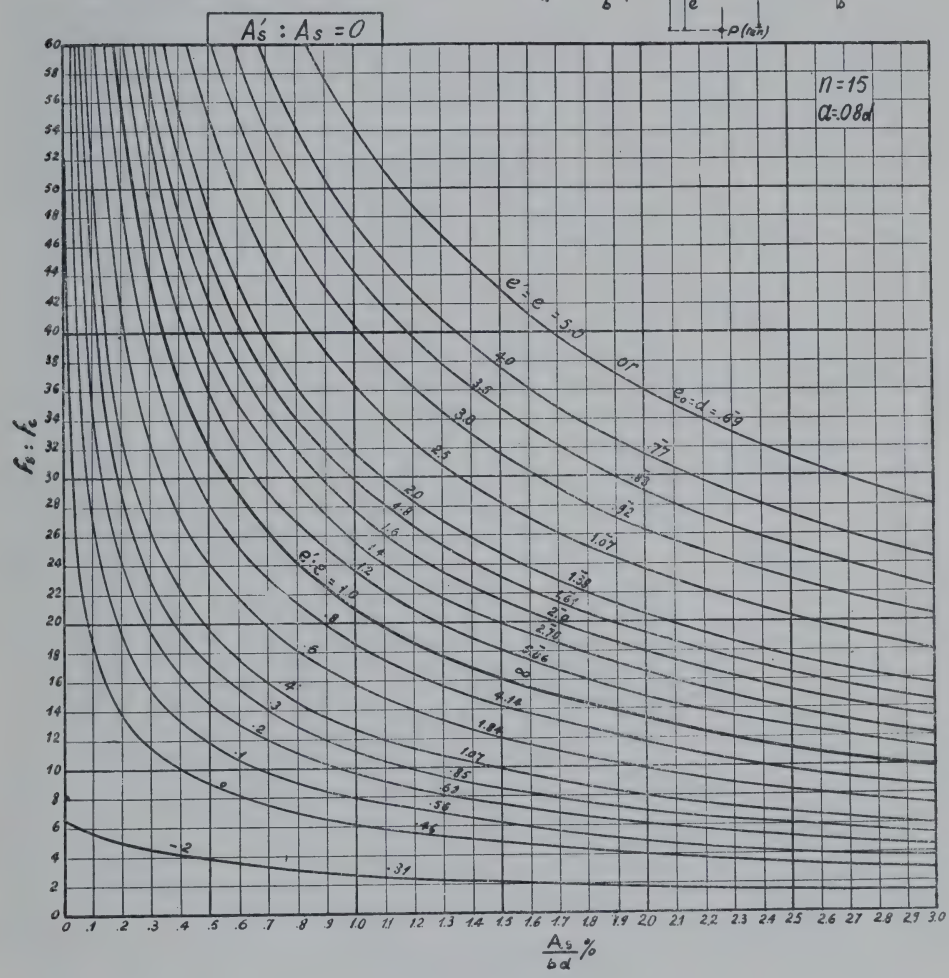
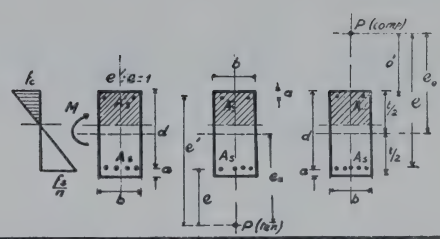
$$d = K_1 \sqrt{M_s} ; K_1 = \frac{C}{\sqrt{k}} \quad A_s = \frac{M_s}{K_2 d} - \frac{P}{F_s} ; K_2 = c' f_s \quad z = sd \quad y_{ct} = c'd$$

$M_s = \text{Moment about } A_s \text{ \& } M_s = \text{Moment about } C$



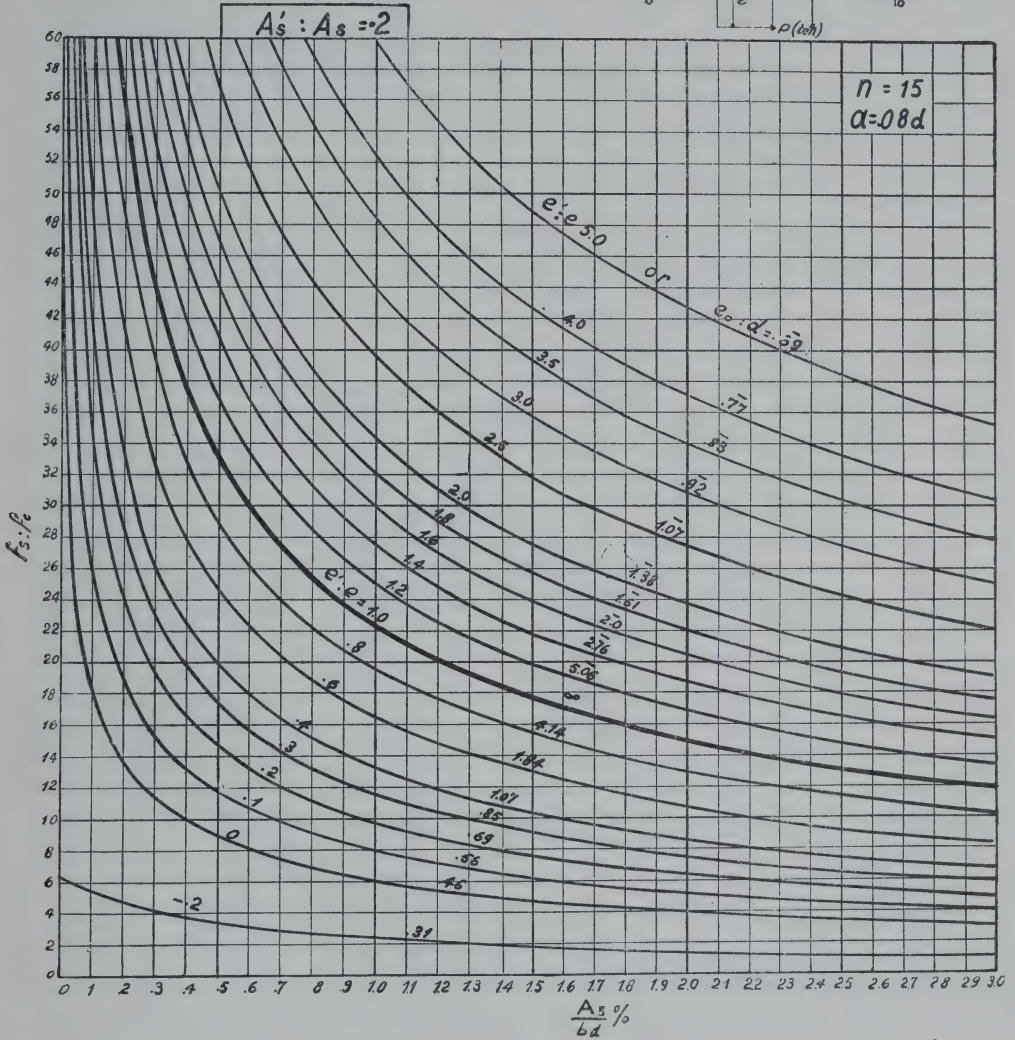
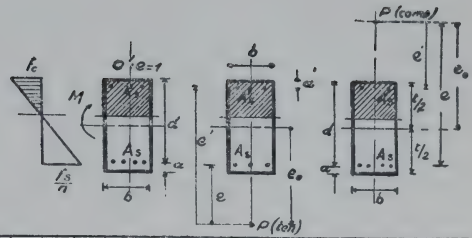
Curve 11

Check of Stresses of Rectangular Sections
Under Bending, Eccentric Comp. or Tension



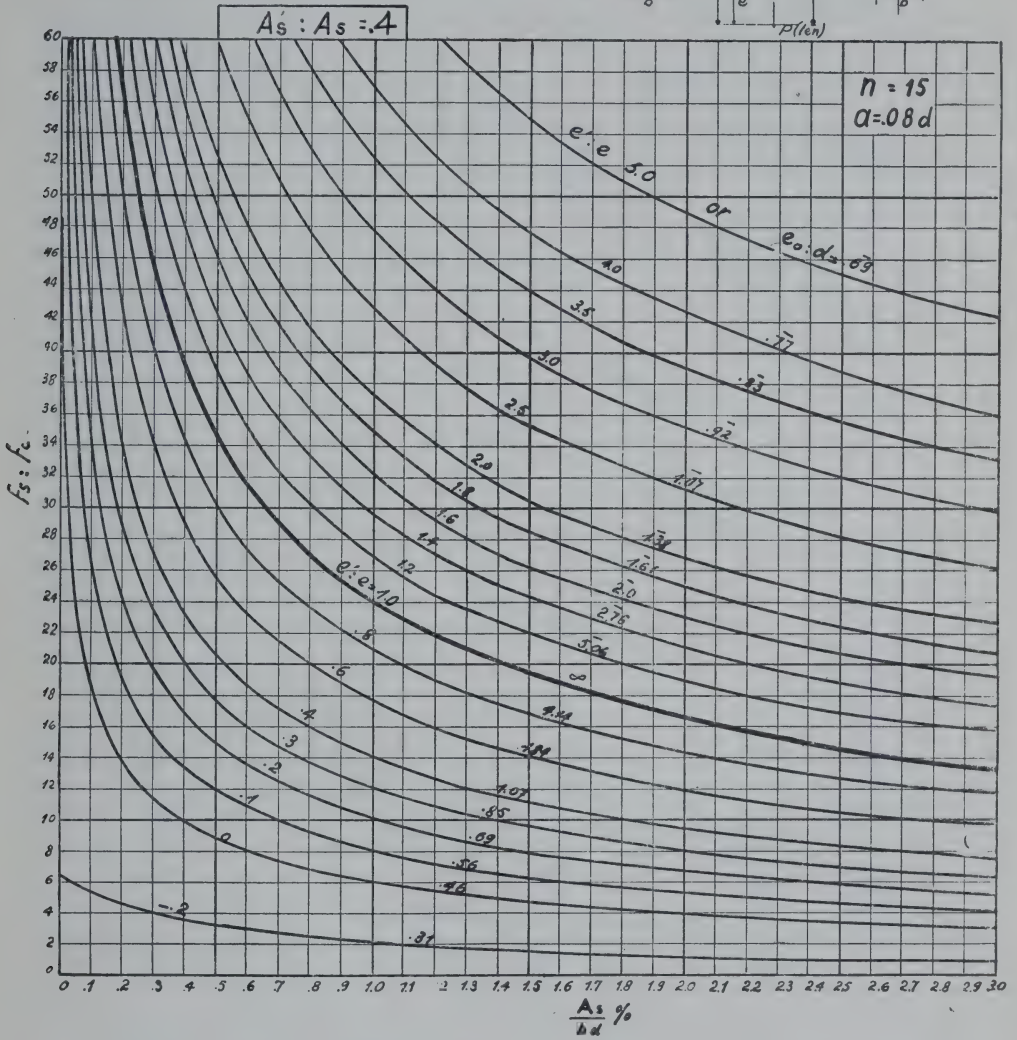
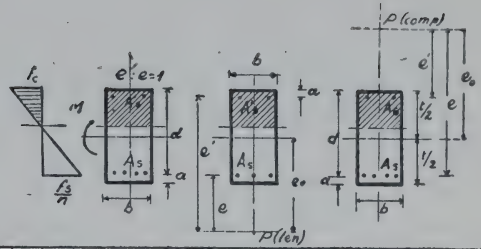
Curve 12

Check of Stresses of Rectangular Sections
Under Bending, Eccentric Comp. or Tension



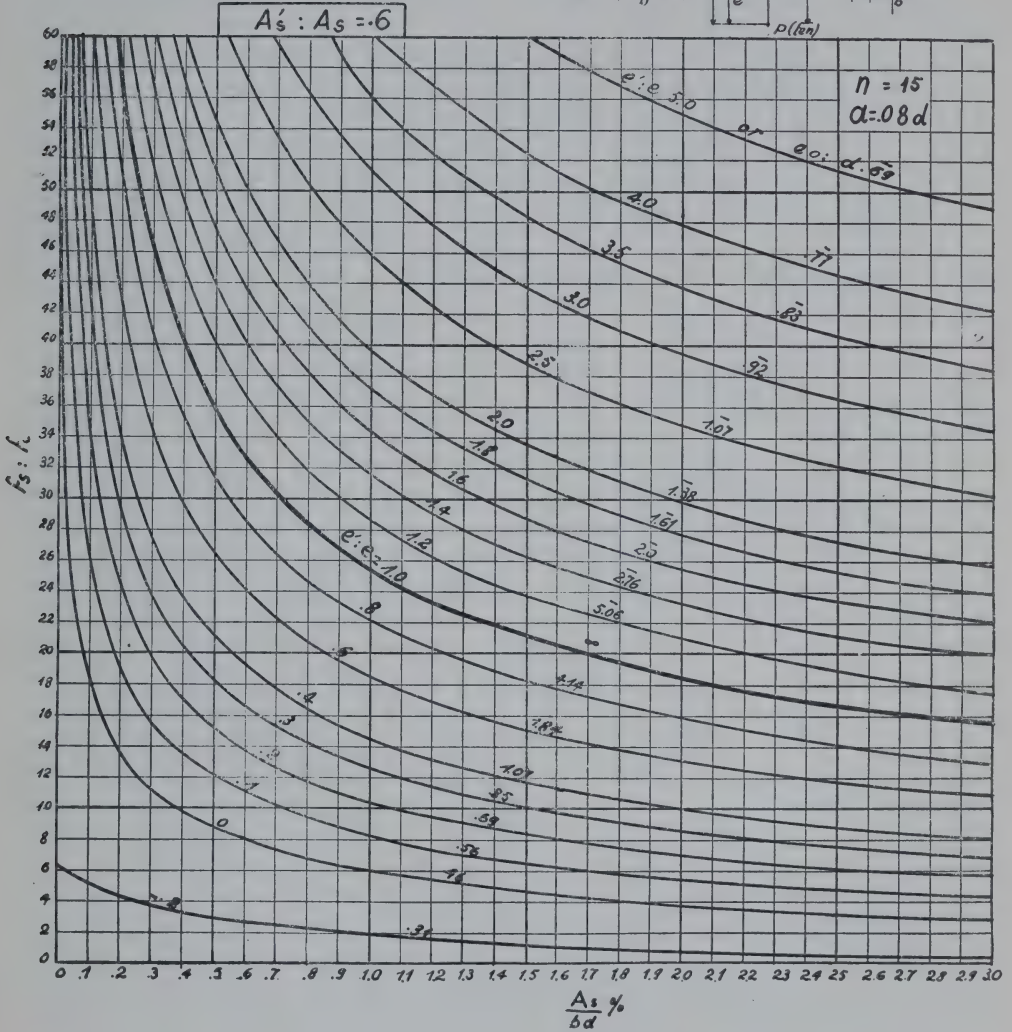
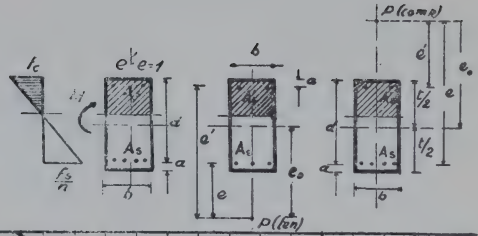
Curve 13

Check of Stresses of Rectangular Sections
Under Bending, Eccentric Comp. or Tension



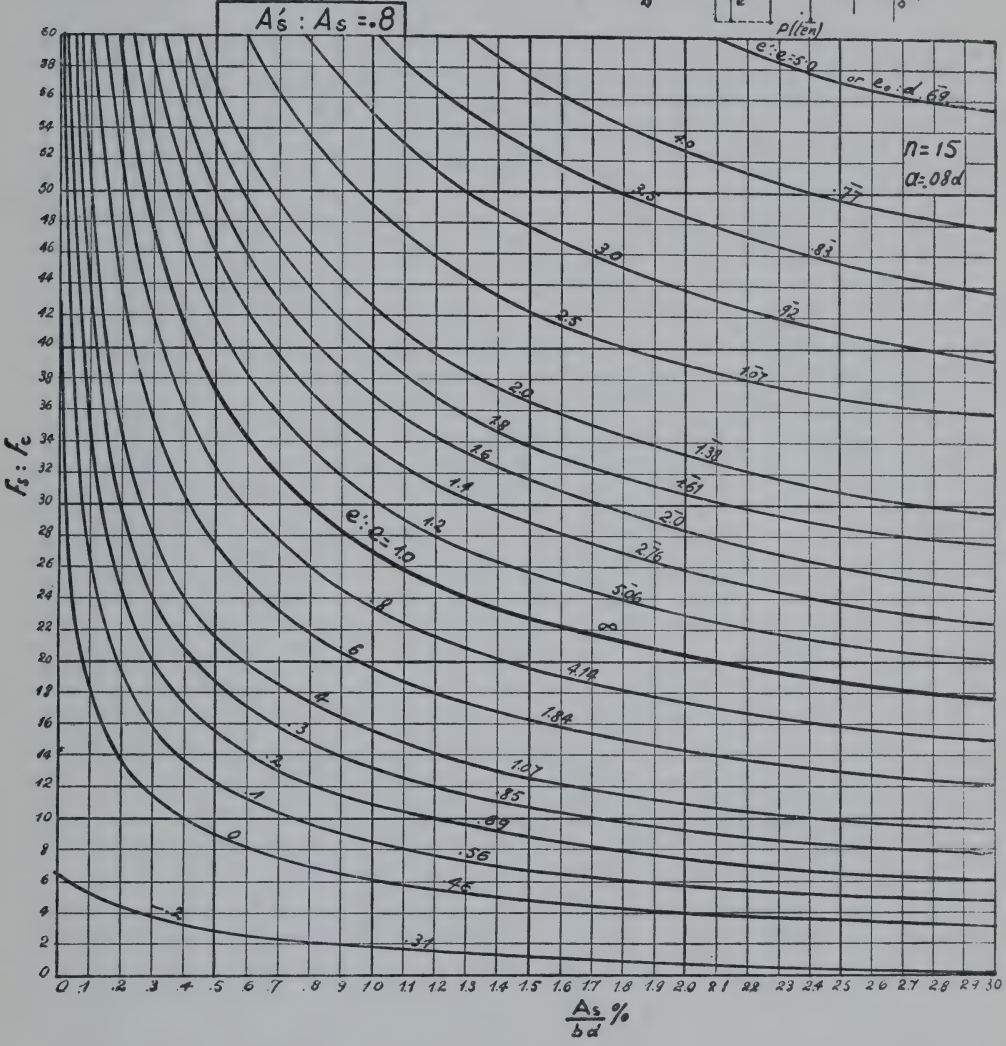
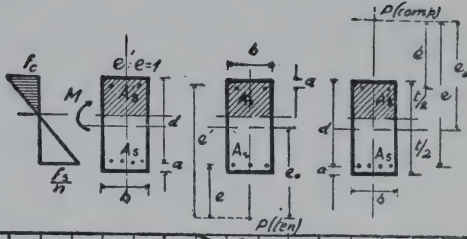
Curve 14

Check of Stresses of Rectangular Sections
Under Bending, Eccentric Comp. or Tension



Curve 15

Check of Stresses of Rectangular Sections
Under Bending, Eccentric Comp. or Tension



Curve 16

Check of Stresses of Rectangular Sections
Under Bending, Eccentric Comp. or Tension

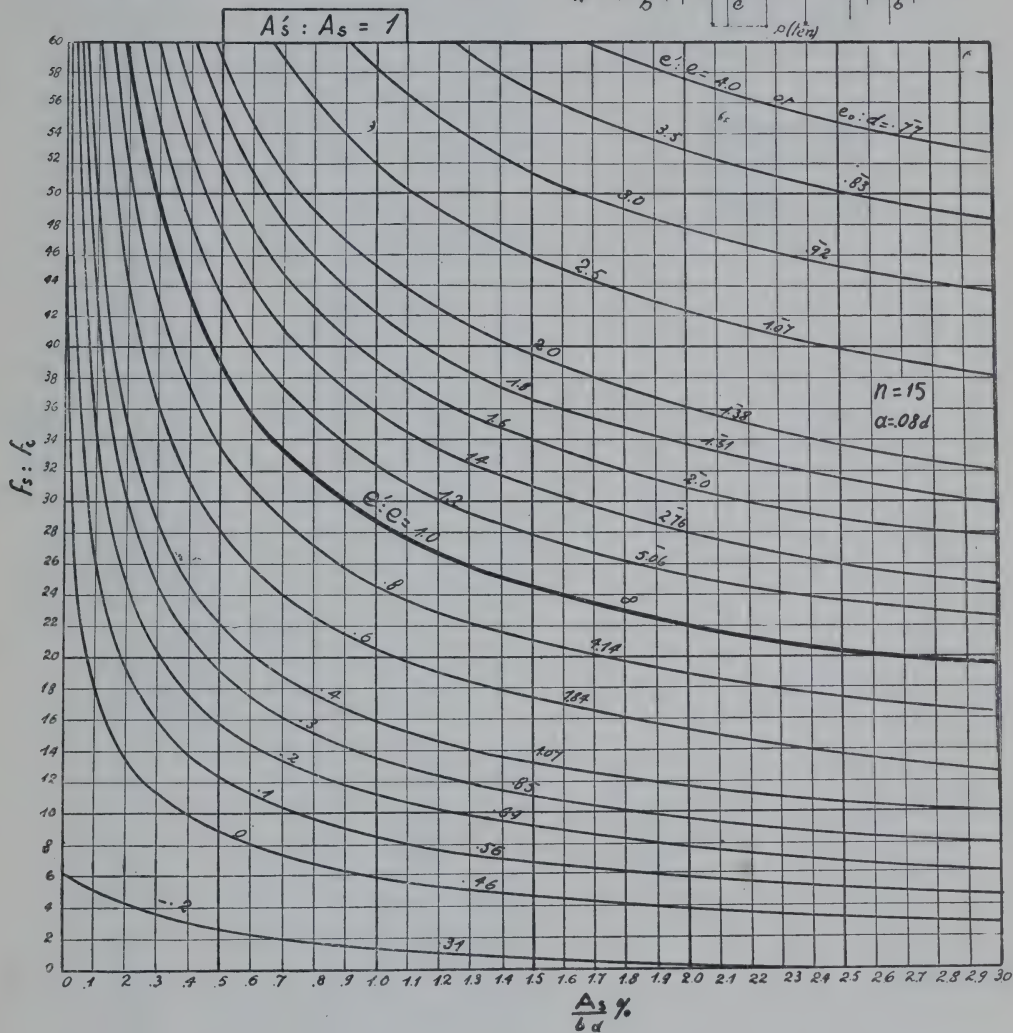
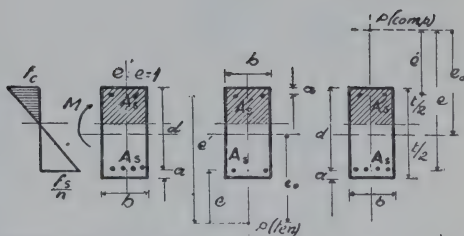


Table 1

Rectangular Sections under Bending, Eccentric Compression, or Eccentric Tension

Given d, f_c, f_s - Required A_s & A'_s

($n = 15$)

($\alpha = .08d$)

$R = \frac{f_s}{f_c}$	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
C_1	7.94	8.06	8.16	8.25	8.35	8.46	8.57	8.67	8.76	8.88	9.00	9.12	9.25	9.40	9.53	9.66	9.80	9.95	10.1	10.25	10.42	10.55	10.7	10.9	11.1
C_2	2.47	2.34	2.21	2.10	2.00	1.91	1.83	1.76	1.69	1.63	1.58	1.53	1.48	1.44	1.41	1.38	1.35	1.32	1.29	1.27	1.25	1.25	1.21	1.20	1.18
C_3	5.43	2.41	1.81	1.355	1.085	0.905	0.775	0.677	0.603	0.542	0.492	0.452	0.417	0.376	0.361	0.340	0.319	0.302	0.286	0.270	0.258	0.246	0.236	0.226	0.217
C_4	5.1	1.86	1.02	0.608	0.391	0.264	0.184	0.133	0.098	0.074	0.055	0.042	0.033	0.024	0.019	0.014	0.011	0.008	0.006	0.004	0.003	0.001	-	-	-

$$u' \% = C_1 \left(\frac{M_s}{bd^2 f_c} \right) - C_2$$

$$u \% = C_3 \left(\frac{M'_s}{bd^2 f_c} \right) + C_4$$

$$u' = \frac{A'_s}{bd}$$

$$u = \frac{A_s}{bd}$$

M_s = moment about tensile reinf.

M'_s = moment about compr. reinf.

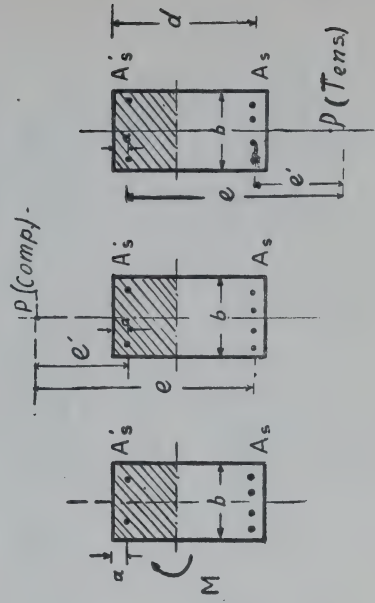
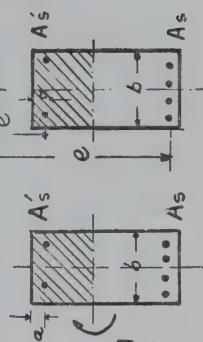


Table: 2: Value of f_s Giving Minimum Reinforcement

($\alpha = .08 d$)

$\frac{M_s}{bd^2f_c}$	$\frac{M'_s}{bd^2f_c}$															
0	.126	.117	.098	.075	.053	.030	.009	.013	.031	.048	.066	.094	.115	.133	.146	.154
.10	.126	.117	.097	.074	.050	.025	.002	.023	.044	.065	.087	.126	.162	.198	.232	.265
.20	.126	.117	.097	.072	.047	.020	.005	.033	.058	.082	.109	.159	.210	.263	.318	.378
.30	.126	.117	.096	.071	.044	.016	.012	.042	.071	.099	.131	.192	.257	.330	.404	.49
.40	.126	.117	.095	.069	.041	.011	.019	.052	.084	.116	.152	.224	.304	.396	.49	-
.50	.126	.116	.094	.067	.038	.006	.026	.062	.097	.133	.173	.256	.350	.461	-	-
.60	.126	.116	.094	.066	.035	.001	.033	.072	.110	.150	.194	.289	.396	-	-	-
.70	.126	.116	.093	.064	.032	.003	.040	.082	.123	.167	.216	.322	.444	-	-	-
.80	.126	.116	.092	.062	.028	.008	.047	.091	.136	.184	.238	.354	.491	-	-	-
$R = \frac{f_s}{f_c}$	0	2	4	6	8	10	12	14	16	18	20	24	28	32	36	40



M_s = moment about tensile reinf.
 M'_s = moment about compr. reinf.

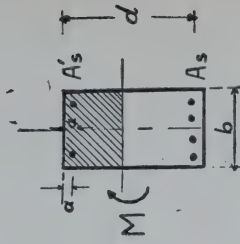
Table 3: Dimensioning and Check of Stresses of Rectangular Sections under Simple Bending

(a): Values of w %

$R = f_s / f_c$	12	16	20	24	28	32	36	40	44	48	52	56	60
$\alpha = 0$	2.32	1.51	1.07	.80	.62	.50	.41	.34	.29	.25	.215	.19	.167
$\alpha = .2$	2.95	1.79	1.22	.89	.68	.535	.435	.36	.303	.258	.224	.195	.172
$\alpha = .4$	4.05	2.20	1.42	1.00	.75	.58	.465	.382	.32	.27	.233	.203	.177
$\alpha = .6$	6.45	2.85	1.69	1.14	.83	.63	.50	.405	.34	.284	.243	.21	.183
$\alpha = .8$	-	4.04	2.04	1.33	.93	.69	.54	.433	.354	.297	.253	.218	.190
$\alpha = 1.0$	-	6.95	2.75	1.59	1.06	.767	.585	.465	.375	.313	.264	.224	.196

($n = 15$)

($q = .08d$)



(b) Values of c

$R = \frac{f_s}{f_c}$	12	16	20	24	28	32	36	40	44	48	52	56	60
$\alpha = 0$	2.1	2.22	2.33	2.44	2.55	2.65	2.75	2.84	2.94	3.04	3.12	3.20	3.28
$\alpha = .2$	1.84	2.02	2.17	2.31	2.44	2.55	2.66	2.76	2.86	2.96	3.05	3.14	3.23
$\alpha = .4$	1.57	1.81	2.00	2.17	2.32	2.45	2.57	2.68	2.80	2.90	3.00	3.08	3.17
$\alpha = .6$	1.24	1.62	1.83	2.03	2.19	2.35	2.48	2.60	2.71	2.83	2.93	3.03	3.13
$\alpha = .8$	-	1.32	1.65	1.87	2.07	2.24	2.39	2.51	2.64	2.76	2.87	2.97	3.07
$\alpha = 1.0$	-	1.00	1.42	1.71	1.93	2.12	2.29	2.43	2.57	2.69	2.81	2.92	3.03

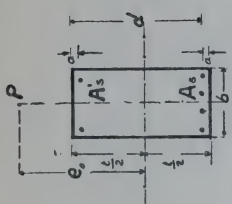
$$w = \frac{A_s}{bd}$$

$$\alpha = \frac{A_s'}{A_s}$$

$$d = k_1 \sqrt{\frac{M}{b}}$$

$$k_1 = \frac{c}{\sqrt{f_c}}$$

$$f_c = \frac{c^2 M}{bd^2}$$



$$d = \left(\frac{P}{b f_c} \right)^{\frac{1}{2}}$$

$$A_s = u b d$$

Table 4

Dimensioning of Rectangular Sections
under Eccentric Compression or Tension

$$A'_s A_s = 0$$

$$(\alpha = 0.8d)$$

$R = \frac{l_z}{f_c}$	Equation of γ	Values of $\frac{bde_s}{P}$															$R = \frac{l_z}{f_c}$	
		$u=0$	$u=2$	$u=4$	$u=6$	$u=8$	$u=10$	$u=12$	$u=14$	$u=16$	$u=18$	$u=20$	$u=22$	$u=24$	$u=26$	$u=28$		
10	300-10 u	1.135	1.142	1.179	1.225	1.287	3.7	4.84	6.5	8.92	12.8	19.4	31.8	58.9	141.0	577	∞	
12	278-12 u	1.275	1.169	2.28	3.10	4.32	6.15	9.18	14.5	25.2	51.3	145.	41.2	2400	2360	90.9	50.5	
14	258-14 u	1.42	2.01	2.96	4.40	6.85	11.4	21.2	48	172.	585	5050	1130	52.5	32.1	22.2	16.7	
16	242-16 u	1.57	2.42	3.85	6.35	11.6	24.5	71	602	1130	1460	510	273	19.7	14.6	11.5	9.70	
18	227-18 u	1.71	2.86	5.02	9.65	22.4	77.5	7550	364.	742	34.6	21.3	14.9	11.3	9.05	7.78	6.86	
20	214-20 u	1.85	3.41	6.75	15.9	54.4	9050	324.	628	29.2	18.1	12.8	9.7	7.77	6.42	5.52	4.80	
22	202-22 u	2.00	4.09	9.42	29.2	2410	616.	67.5	28.5	16.9	11.7	8.9	7.1	5.88	4.98	4.34	3.83	
24	192-24 u	2.14	4.88	13.3	630	∞	91.5	32.2	17.7	11.7	8.62	6.81	5.59	4.70	4.09	3.62	3.24	
26	183-26 u	2.28	5.87	20.0	204.	312	46.	20.2	12.6	8.85	6.85	5.5	4.58	3.94	3.46	3.07	2.75	
28	174-28 u	2.43	7.12	32.5	4190	588	26.7	14.3	9.6	7.10	5.69	4.6	3.91	3.38	3.00	2.67	2.41	
30	166-30 u	2.58	8.8	59.8	850	453	18.5	11.1	7.7	5.90	4.74	3.95	3.38	2.97	2.64	2.37	2.16	
32	159-32 u	2.73	10.9	133.	1710	284	14.2	8.98	6.48	5.03	4.12	3.48	3.02	2.65	2.36	2.15	1.94	
34	153-34 u	2.86	13.6	196.	79.5	212	11.3	7.50	5.52	4.42	3.66	3.10	2.70	2.38	2.15	1.95	1.79	
36	147-36 u	3.00	17.4	49500	47.6	165	9.38	6.46	4.91	3.94	3.29	2.82	2.46	2.18	1.97	1.79	1.63	
38	141-38 u	3.16	23.0	1180	32.8	133	8.00	5.65	4.36	3.54	2.98	2.57	2.26	2.00	1.79	1.65	1.51	
40	136-40 u	3.37	31.1	272.	24.6	11.1	7.00	5.05	3.92	3.22	2.73	2.36	2.08	1.85	1.68	1.52	1.41	
42	131-42 u	3.45	44.2	125	19.6	9.6	6.2	4.55	3.88	3.27	2.52	2.18	1.92	1.72	1.57	1.43	1.31	
44	127-44 u	3.57	64.8	76.5	16.3	8.45	5.58	4.16	3.52	2.75	2.34	2.04	1.80	1.62	1.47	1.35	1.24	
46	123-46 u	3.74	101.	53.0	13.9	7.51	5.08	3.84	3.07	2.65	2.18	1.90	1.69	1.52	1.38	1.27	1.17	
48	119-48 u	3.92	189.	39.6	12.1	6.82	4.7	3.57	2.87	2.40	2.07	1.80	1.60	1.43	1.31	1.21	1.13	
		Ecc. Comp.															Eccentric Tension	

Table 5

Dimensioning of Rectangular Sections
under Eccentric Compression or Tension

$$d = \left(\frac{P}{b f_c} \right)^{\frac{1}{3}}$$

$$A_s = \mu b d$$

$$A_s A_s = .2$$

$$(\alpha = .08d)$$

$R = \frac{f_s}{f_c}$	Equation of γ	Values of $\frac{b f_c d}{P}$																$R = \frac{f_s}{f_c}$
		$\mu = 0$	$\mu = .2$	$\mu = .4$	$\mu = .6$	$\mu = .8$	$\mu = 1.0$	$\mu = 1.2$	$\mu = 1.4$	$\mu = 1.6$	$\mu = 1.8$	$\mu = 2.0$	$\mu = 2.2$	$\mu = 2.4$	$\mu = 2.6$	$\mu = 2.8$	$\mu = 3.0$	
10	$300 - 7.4 \mu$	4.135	4.39	4.71	2.07	2.55	3.42	3.84	4.75	5.40	7.50	9.95	12.2	16.2	22.0	30.4	45.1	10
12	$278 - 9.43 \mu$	4.275	4.66	2.15	2.79	3.71	4.87	6.56	9.01	12.5	18.7	29.2	48.8	95.1	249	1470	11700	12
14	$258 - 11.46 \mu$	4.42	4.98	2.75	3.94	5.69	8.35	12.9	20.9	38.6	85.2	294	7280	1010	204	921	54.2	14
16	$242 - 13.5 \mu$	4.57	2.36	3.59	5.51	8.85	15.5	30.3	75.0	337	24000	366	109	55.0	348	245	18.6	16
18	$227 - 15.53 \mu$	4.71	2.78	4.63	8.05	15.5	35.3	1190	2200	535	109	50.0	30.3	21.0	15.9	26.1	10.3	18
20	$214 - 17.56 \mu$	4.85	3.30	6.10	12.4	30.8	1300	23200	253	6.95	35.1	22.2	15.8	12.2	9.75	8.15	6.95	20
22	$202 - 19.59 \mu$	2.00	3.93	8.25	21.1	84.0	5370	226	52.5	28.2	18.5	13.3	10.3	8.30	6.90	5.91	5.18	22
24	$192 - 21.62 \mu$	2.14	4.64	11.4	39.4	486.0	3560	62.1	58.5	17.5	12.2	9.32	7.51	6.31	5.31	4.66	4.14	24
26	$188 - 23.66 \mu$	2.28	5.58	16.7	92.0	5180	880	32.0	18.0	12.1	9.05	7.21	5.88	5.00	4.38	3.83	3.43	26
28	$174 - 25.69 \mu$	2.43	6.71	25.5	392	2080	418	20.0	42.6	9.15	7.10	5.72	4.82	4.15	3.66	3.23	2.93	28
30	$166 - 27.72 \mu$	2.58	8.2	43.0	16100	770	25.6	14.5	9.75	7.29	5.82	4.80	4.08	3.54	3.13	2.83	2.54	30
32	$159 - 29.75 \mu$	2.73	10.1	82.5	451	41.8	18.2	11.1	7.89	6.09	4.91	4.10	3.52	3.08	2.74	2.47	2.26	32
34	$153 - 31.78 \mu$	2.86	12.6	198	1470	226	14.3	9.15	6.65	5.25	4.20	3.62	3.13	2.76	2.46	2.23	2.03	34
36	$147 - 33.82 \mu$	3.00	15.9	110	70.5	207	11.3	7.65	5.72	4.52	3.78	3.21	2.80	2.47	2.22	2.01	1.84	36
38	$141 - 35.85 \mu$	3.16	20.9	15500	42.9	16.2	9.37	6.55	5.02	4.08	3.36	2.89	2.53	2.26	2.02	1.84	1.69	38
40	$136 - 37.88 \mu$	3.31	27.6	678.0	31.4	13.2	8.15	5.80	4.46	3.64	3.06	2.64	2.32	2.06	1.87	1.70	1.56	40
42	$131 - 39.91 \mu$	3.45	38.2	2120	24.2	11.2	7.01	5.45	4.02	3.31	2.79	2.42	2.13	1.90	1.73	1.57	1.45	42
44	$127 - 41.94 \mu$	3.57	54.2	108	19.5	9.7	6.30	4.66	3.65	3.03	2.56	2.23	1.97	1.77	1.61	1.47	1.36	44
46	$123 - 43.98 \mu$	3.74	86	68.5	16.3	8.5	5.70	4.26	3.36	2.80	2.40	2.08	1.85	1.67	1.51	1.39	1.27	46
48	$119 - 46.00 \mu$	3.92	1410	44.5	13.8	7.6	5.21	3.92	3.14	2.62	2.25	1.96	1.73	1.58	1.42	1.31	1.20	48
		Eccentric Tension																
		Ecc. Comp.																

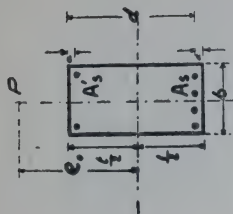


Table 6

Dimensioning of Rectangular Sections
under Eccentric Compression or Tension

(n=15)

(α=.08d)

A's: A_s=4

$$d = \left(\frac{P}{b f_c} \right) \frac{1}{\phi}$$

$$A_s = u b d$$

$R \frac{f_s}{f_c}$	Equation of σ	Values of $\frac{b f e_0}{P}$															$R \frac{f_s}{f_c}$	
		$u=0$	$u=.2$	$u=.4$	$u=.6$	$u=.8$	$u=1.0$	$u=1.2$	$u=1.4$	$u=1.6$	$u=1.8$	$u=2.0$	$u=2.2$	$u=2.4$	$u=2.6$	$u=2.8$		
10	$300-4.8 u$	1.135	1.57	1.65	1.95	2.3	2.70	3.16	3.68	4.27	4.92	5.81	6.80	7.90	9.25	10.9	12.8	10
12	$278-6.86 u$	1.275	1.64	2.07	2.59	3.25	4.03	5.02	6.28	7.96	10.2	12.9	16.9	22.6	30.4	43.1	64.5	12
14	$258-8.93 u$	1.42	1.95	2.63	3.53	4.71	6.2	8.8	12.3	17.8	26.9	43.2	74.5	158.0	478	5310	5670	14
16	$242-10.99 u$	1.57	2.28	3.32	4.80	7.11	10.8	17.1	29.1	52.5	137	585	∞	7100	1980	970	595	16
18	$227-13.06 u$	1.71	2.7	4.26	6.8	11.4	20.9	43.8	122.0	795	4440	2870	1010	55.5	362	266	20.3	18
20	$214-15.12 u$	1.85	3.18	5.55	10.1	20.4	50.0	191	60700	3740	990	50.0	31.5	22.4	16.9	13.5	11.2	20
22	$202-17.18 u$	2.00	3.78	7.39	15.9	42.8	228	18700	200	665	35.2	23.2	16.7	13.0	10.5	8.80	7.58	22
24	$192-19.25 u$	2.14	4.42	9.9	26.8	128	212000	1710	55.1	24.2	19.0	13.9	10.9	8.75	7.43	6.35	5.59	24
26	$183-21.31 u$	2.28	5.38	13.6	32.8	1120	2730	605	28.6	180	12.9	9.90	7.97	6.65	5.71	5.00	4.42	26
28	$174-23.38 u$	2.43	6.41	20.8	44.0	1210	79.5	31.3	18.1	124	945	753	621	5.27	4.60	4.08	3.64	28
30	$166-25.44 u$	2.58	7.75	32.9	98.0	170	40.3	20.1	130	9.4	7.33	6.00	5.05	4.36	3.86	3.42	3.07	30
32	$159-27.50 u$	2.73	9.5	56.8	1870	69.5	25.5	19.6	100	7.55	6.02	5.00	4.26	3.72	3.28	2.96	2.67	32
34	$153-29.57 u$	2.86	11.6	1130	338	41.4	18.6	11.4	8.15	6.30	5.12	4.30	3.70	3.23	2.87	2.58	2.36	34
36	$147-31.63 u$	3.00	14.5	347	1170	27.6	14.2	9.29	6.8	5.35	4.42	3.74	3.25	2.86	2.56	2.32	2.12	36
38	$141-33.7 u$	3.16	18.5	3890	63.5	20.3	11.4	7.75	5.85	4.67	3.87	3.31	2.90	2.57	2.31	2.09	1.92	38
40	$136-35.76 u$	3.31	24.6	2440	40.0	15.9	9.4	6.65	5.05	4.12	3.45	2.97	2.62	2.32	2.08	1.90	1.74	40
42	$131-37.82 u$	3.45	33.5	3700	29.3	13.0	805	5.85	456	3.86	3.13	2.71	2.36	2.13	1.91	1.75	1.59	42
44	$127-39.89 u$	3.57	45.9	163	22.9	11.0	7.15	5.22	4.40	336	2.85	2.46	2.18	1.95	1.76	1.63	1.49	44
46	$123-41.95 u$	3.74	67.5	93.0	18.9	9.6	6.32	4.7	3.73	308	263	2.28	2.02	1.82	1.65	1.51	1.39	46
48	$119-44.01 u$	3.92	1080	63.5	15.9	8.60	5.78	4.32	3.43	2.88	2.45	2.16	1.90	1.71	1.55	1.42	1.31	48
		Ecc. Comp.															Eccentric Tension	

Table 7

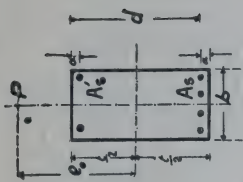
Dimensioning of Rectangular Sections
under Eccentric Compression or Tension

$$A'_s A_s = 6$$

$$(n = 15)$$

$$(a = .08d)$$

$R = \frac{I_s}{F_c}$	Equation of $\bar{\sigma}$	Values of $\frac{bke}{p}$																$R = \frac{I_s}{F_c}$
		$\mu = 0$	$\mu = .2$	$\mu = .4$	$\mu = .6$	$\mu = .8$	$\mu = 1.0$	$\mu = 1.2$	$\mu = 1.4$	$\mu = 1.6$	$\mu = 1.8$	$\mu = 2.0$	$\mu = 2.2$	$\mu = 2.4$	$\mu = 2.6$	$\mu = 2.8$	$\mu = 3.0$	
10	$300 - 22 \mu$	1.135	1.36	1.60	1.83	2.10	2.37	2.68	2.98	3.32	3.66	4.06	4.44	4.86	5.30	5.82	6.35	10
12	$278 - 4.3 \mu$	1.275	1.61	1.98	2.40	2.86	3.41	4.05	4.75	5.55	6.53	7.55	8.85	10.3	12.1	14.1	16.7	12
14	$258 - 6.39 \mu$	1.42	1.91	2.48	3.18	4.05	5.13	6.5	8.2	10.4	13.3	17.3	22.8	30.2	41.5	59.5	900	14
16	$242 - 8.49 \mu$	1.57	2.23	3.11	4.28	5.85	8.05	11.4	16.1	23.3	36.2	59.5	108	243	840	24700	2520	16
18	$227 - 10.58 \mu$	1.71	2.63	3.95	5.90	8.99	14.0	22.8	40.3	82.0	218.0	1430	1400	5.57	189.0	100	64.7	18
20	$214 - 12.68 \mu$	1.85	3.09	5.1	8.4	14.5	27.8	61.5	201.0	236.0	1650	2480	9850	57.9	39.2	286	22.3	20
22	$202 - 14.78 \mu$	2.00	3.66	6.6	12.7	26.8	74.0	390	1000	284	93.4	505	32.5	23.5	18.0	14.7	12.2	22
24	$192 - 16.87 \mu$	2.14	4.28	8.75	19.8	59.5	420	2600	165	62.2	34.9	23.7	17.4	13.6	11.1	9.4	8.10	24
26	$183 - 18.97 \mu$	2.28	5.05	12.0	35.0	204	4760	1480	54.1	30.0	19.9	14.7	11.4	9.35	7.95	6.78	5.95	26
28	$174 - 21.06 \mu$	2.43	6.05	17.0	74.0	562.0	1950	54.2	28.0	18.2	13.2	10.2	8.25	6.95	5.95	5.25	4.62	28
30	$166 - 23.16 \mu$	2.58	7.3	26.0	238	620	690	30.1	18.1	12.7	9.65	7.75	6.43	5.48	4.78	4.23	3.84	30
32	$159 - 25.26 \mu$	2.73	8.9	41.6	3560	1360	375	20.0	13.0	9.65	7.55	6.25	5.25	4.55	3.98	3.55	3.22	32
34	$153 - 27.35 \mu$	2.86	10.9	73.1	1310	63.5	25.0	14.6	10.2	7.7	6.18	5.16	4.40	3.85	3.42	3.05	2.79	34
36	$147 - 29.45 \mu$	3.00	13.5	170	233	38.4	18.1	11.4	8.25	6.40	5.23	4.40	3.80	3.34	2.98	2.67	2.44	36
38	$141 - 31.54 \mu$	3.16	17.1	645	99.0	26.6	14.4	9.35	6.95	5.50	4.52	3.88	3.34	2.94	2.64	2.40	2.19	38
40	$136 - 33.64 \mu$	3.31	24.6	36600	57.2	20.0	11.5	7.9	5.92	4.75	3.98	3.40	2.97	2.64	2.37	2.14	1.97	40
42	$131 - 35.74 \mu$	3.45	28.8	1100	39.2	15.7	9.52	6.7	5.21	4.21	3.54	3.04	2.67	2.37	2.15	1.95	1.79	42
44	$127 - 37.83 \mu$	3.57	39.5	298	28.8	13.0	8.22	5.95	4.62	3.8	3.19	2.76	2.44	2.17	1.97	1.79	1.65	44
46	$123 - 39.93 \mu$	3.74	56.0	136	22.5	11.0	7.2	5.3	4.17	3.42	2.91	2.52	2.22	2.00	1.82	1.66	1.52	46
48	$119 - 42.02 \mu$	3.92	86.2	83.5	18.3	9.6	6.47	4.80	3.81	3.15	2.70	2.35	2.08	1.87	1.69	1.55	1.42	48
Ecc. Comp.		Eccentric Tension																



$$d = \left(\frac{P}{b F_c} \right) \frac{1}{\bar{\sigma}}$$

$$A_s = \mu b d$$

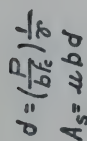
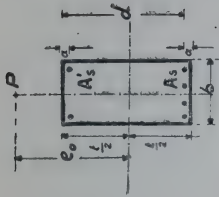


Table 8

Dimensioning of Rectangular Sections Under Eccentric Compression or Tension

[illegible]



$$d = \left(\frac{P}{b f_c} \right) \frac{1}{\gamma}$$

$$A_s = w b d$$

Table 9

Dimensioning of Rectangular Sections
under Eccentric Compression or Tension

$$A'_s A_s = I$$

$$(\alpha = 0.8d)$$

$$(n=15)$$

$R_c \frac{f_c}{f_c}$	Equation of γ	Values of $\frac{b h e_s}{P}$															$R_c \frac{f_c}{f_c}$
		$u=0$	$u=2$	$u=4$	$u=6$	$u=8$	$u=10$	$u=12$	$u=14$	$u=16$	$u=18$	$u=20$	$u=22$	$u=24$	$u=26$	$u=28$	
10	$300 + 3.0 u$	1.135	1.315	1.485	1.64	1.78	1.905	2.03	2.14	2.24	2.32	2.42	2.50	2.57	2.64	2.70	10
12	$278 + 84 u$	1.275	1.55	1.825	2.082	2.34	2.60	2.84	3.08	3.31	3.55	3.76	3.99	4.19	4.39	4.59	12
14	$258 + 132 u$	1.42	1.83	2.24	2.70	3.15	3.62	4.13	4.63	5.16	5.74	6.32	6.95	7.63	8.23	8.96	14
16	$242 + 348 u$	1.57	2.13	2.78	3.49	4.29	5.20	6.22	7.38	8.71	10.2	11.95	14.00	16.3	18.7	21.7	16
18	$227 + 564 u$	1.71	2.49	3.45	4.61	6.02	7.80	10.10	12.95	16.60	21.7	28.2	37.30	50.0	70.3	100.8	18
20	$214 + 78 u$	1.85	2.92	4.32	6.21	8.80	12.6	18.20	26.5	40.5	64.0	115.0	235.0	604	388.0	315.00	20
22	$202 + 986 u$	2.00	3.41	5.51	8.70	13.9	22.9	40.0	75.8	180.0	677.0	4380.0	1530	3590	1670	997	22
24	$192 + 12.12 u$	2.14	3.97	7.00	12.55	23.4	48.4	125.5	64.0	88200	593.0	182.0	91.0	57.7	41.1	31.3	24
26	$183 + 14.28 u$	2.28	4.67	9.20	19.15	45.2	156.0	197.0	116.0	187.0	83.5	49.6	33.7	25.0	19.8	16.2	26
28	$174 + 16.44 u$	2.43	5.53	12.6	32.4	124.0	256.0	619.0	121.0	56.4	34.5	24.2	18.5	14.7	12.1	10.3	28
30	$166 + 18.60 u$	2.58	6.56	17.4	63.6	172.0	700.0	108.3	48.0	28.5	19.9	15.1	12.1	10.0	8.47	7.38	30
32	$159 + 20.76 u$	2.73	7.93	25.7	183.0	367.0	125.0	47.3	26.4	18.1	13.4	10.6	8.73	7.41	6.28	5.65	32
34	$153 + 22.92 u$	2.86	9.46	40.3	849.0	271.0	58.2	28.3	18.0	13.0	10.0	8.25	6.83	5.88	5.14	4.58	34
36	$147 + 25.08 u$	3.00	11.5	68.5	2490.	102.0	35.4	19.9	13.3	9.96	7.96	6.56	5.58	4.85	4.28	3.83	36
38	$141 + 27.24 u$	3.16	14.3	149.0	4490	52.7	23.7	14.5	10.35	7.96	6.46	5.42	4.65	4.07	3.63	3.27	38
40	$136 + 29.4 u$	3.31	18.1	4760	142.0	33.9	17.4	11.4	8.40	6.60	5.42	4.59	4.0	3.52	3.15	2.84	40
42	$131 + 31.56 u$	3.45	23.0	6240	168.8	24.9	13.9	9.44	7.10	5.64	4.68	4.00	3.49	3.10	2.79	2.53	42
44	$127 + 33.72 u$	3.57	30.0	2570	47.5	19.1	11.3	7.94	6.09	4.90	4.11	3.53	3.10	2.75	2.48	2.25	44
46	$123 + 35.88 u$	3.74	41.4	4450	55.1	15.4	9.61	6.88	5.34	4.35	3.68	3.17	2.79	2.49	2.25	2.04	46
48	$119 + 38.04 u$	3.92	59.2	1180	26.8	12.9	8.32	6.09	4.77	3.92	3.32	2.89	2.54	2.28	2.06	1.88	48

Ecc. Comp.

Eccentric Tension

Table 10

Dimensioning of Sections with no compression reinforcement under Bending, Eccentric Compression, or Eccentric Tension

$$d = K_1 \sqrt{\frac{M_s}{b}}$$

$$\text{or } d = K_1 \left[\pm r + \sqrt{r^2 + \frac{M}{b}} \right]$$

$$A_s = \frac{M_{s1}}{K_2 d}$$

$$\text{or } A_s = \frac{M_s}{K_2 d} + \frac{P}{f_s}$$

Where $K_1 = \frac{c}{\sqrt{f_c}}$

$$r = \frac{P}{b} \times \frac{K K_1}{2}$$

+ for eccentric Comp.
- for eccentric Tension

- for eccentric Compression
+ for eccentric Tension

$$K_2 = c' f_s$$

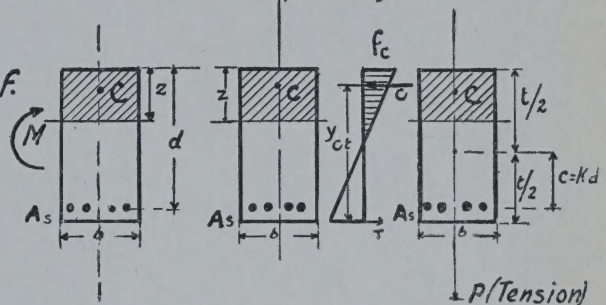
$P(\text{comp})$

M_s = Moment abt. tensile Reinf.

M_{s1} = Moment abt. centre of
Compression C.

$$Z = s d$$

$$y_{cr} = c' d$$



(n = 15)

$\frac{f_s}{f_c}$	10	12	14	16	18	20	22	24	26	28	30	32	34
S	.600	.556	.517	.484	.455	.429	.405	.385	.366	.349	.333	.319	.306
C	2.04	2.10	2.16	2.22	2.28	2.33	2.38	2.44	2.49	2.55	2.60	2.65	2.70
C'	.800	.815	.828	.839	.848	.857	.865	.872	.878	.884	.889	.894	.898

$\frac{f_s}{f_c}$	36	38	40	42	44	46	48	50	52	54	56	58	60
S	.294	.283	.273	.263	.254	.246	.238	.231	.224	.217	.211	.205	.200
C	2.75	2.80	2.84	2.89	2.94	2.98	3.04	3.08	3.12	3.16	3.20	3.24	3.28
C'	.902	.906	.909	.912	.915	.918	.921	.923	.925	.927	.929	.931	.933

